A LOW COMPLEXITY SCHEDULING FOR DOWNLINK OF OFDMA SYSTEM WITH PROPORTIONAL RESOURCE ALLOCATION

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Abstract—The base station allocation of subcarriers and power to each user to maximize the sum of user data rates, subject to constraints on total power, bit error rate, and proportionality among user data rates. Previous allocation methods have been iterative nonlinear methods suitable for offline optimization. In the special high subchannel SNR case, an iterative root-finding method has linear-time complexity in the number of users and $N \log N$ complexity in the number of subchannels. We propose a non-iterative method that is made possible by our relaxation of strict user rate proportionality constraints. The proposed method waives the restriction of high subchannel SNR, has significantly lower complexity, and in simulation, yields higher user data rates.

Keywords—orthogonal frequency division multiplexing (OFDMA), cyclic prefix

I. INTRODUCTION

OFDMA, also referred to as Multiuser-OFDM [1], is being considered as a modulation and multiple access method for 4th generation wireless networks [2]. OFDMA is an extension of Orthogonal Frequency Division Multiplexing (OFDM), which is currently the modulation of choice for high speed data access systems such as IEEE 802.11a/g wireless LAN [3] and IEEE 802.16a fixed wireless broadband access [4] systems. OFDM systems divide a broadband channel into many narrowband subchannels. Each subchannel carries a quadrature amplitude modulated (QAM) signal. The subcarriers are combined in a computationally efficient manner by means of an inverse fast Fourier transform (IFFT) in the transmitter. Each complex-valued IFFT input is obtained from a QAM constellation mapping (lookup table). The IFFT outputs form the transmitted symbol. Before transmission, a cyclic prefix is pretended to the symbol. The receiver performs the dual operations of cyclic prefix (CP) removal, FFT, and QAM demapping.

In [5], the margin-adaptive resource allocation problem was tackled, wherein an iterative subcarrier and power allocation algorithm was proposed to minimize the total transmit power given a set of fixed user data rates and bit error rate (BER) requirements. In [6], the rate-adaptive problem was investigated, wherein the objective was to maximize the total data rate over all users subject to power and BER constraints. It was shown in [6] that in order to maximize the total capacity, each subcarrier should be allocated to the user with the best gain on it, and the power should be allocated using the waterfilling algorithm across the subcarriers. However, no fairness among the users was considered in [6]. This problem was partially addressed in [7] by ensuring that each user would be able to transmit at a minimum rate, and also in [8] by incorporating a notion of fairness in the resource allocation through maximizing the minimum user’s data rate.

In [9], the fairness was extended to incorporate varying priorities. Instead of maximizing the minimum user’s capacity, the total capacity was maximized subject to user rate proportionality constraints. This is very useful for service level differentiation, which allows for flexible billing mechanisms for different classes of users.

This paper extends the work in [9] by developing a subcarrier allocation scheme that linearizes the power allocation problem while achieving approximate rate proportionality. The resulting power allocation problem is thus reduced to a solution to simultaneous linear equations. In simulation, the proposed algorithm achieves a total capacity that is consistently higher than the previous work, requires significantly less computation, while achieving acceptable rate proportionality.

II. SYSTEM MODEL

The block diagram for the downlink of a typical OFDMA system is shown in Fig. 1. At the base station transmitter, the bits for each of the different $K$ users are allocated to the $N$ subcarriers, and each subcarrier $n$ ($1 \leq n \leq N$) of user $k$ ($1 \leq k \leq K$) is assigned a power $p_{k,n}$. It is assumed that subcarriers are not shared by different users. Each of the user’s bits are then modulated into $N M$- level QAM symbols, which are subsequently combined using the IFFT into an OFDMA symbol. This is then transmitted through a slowly time-varying, frequency-selective Rayleigh channel with a bandwidth $B$. The subcarrier allocation is made known to all the users through a control channel; hence each user needs only to decode the bits on
their assigned subcarriers.

It is assumed that each user experiences independent fading and the channel gain of user \( k \) in subcarrier \( n \) is denoted as \( g_{k,n} \), with additive white gaussian noise (AWGN) \( \sigma^2 = N_0 \beta \) where \( N_0 \) is the noise power spectral density. The corresponding subchannel signal-to-noise ratio (SNR) is thus denoted as \( h_{k,n} = \frac{g_{k,n}}{\sigma^2} \) and the \( k^{th} \) users received SNR on subcarrier \( n \) is \( Y_{k,n} = p_{k,n} h_{k,n} \). The slowly time-varying assumption is crucial since it is also assumed that each user is able to estimate the channel perfectly and these estimates are made known to the transmitter via a dedicated feedback channel. These channel estimates are then used as input to the resource allocation algorithms.

In order that the BER constraints are met, the effective SNR has to be adjusted accordingly. The BER of a square M-level QAM with Gray bit mapping as a function of received SNR \( Y_{k,n} \) and number of bits \( r_{k,n} \) can be approximated to within 1 dB for \( r_{k,n} \geq 4 \) and BER \( \leq 10^{-3} \).

\[
\text{BER}_{QAM}(Y_{k,m}) \approx 0.2 \exp \left[ -\frac{1.6 \Gamma}{r_{k,n}-1} \right]
\]

Solving for \( r_{k,n} \), we have

\[
r_{k,n} = \log_2 \left( 1 + \frac{Y_{k,n}}{\Gamma} \right) = \log_2 \left( 1 + p_{k,n} H_{k,n} \right)
\]

Where \( \Gamma \approx -\frac{\ln(SNR)}{1.6} \) is a constant SNR gap, and \( H_{k,n} = \frac{h_{k,n}}{\Gamma} \) is the effective subchannel SNR.

The objective of the resource allocation is

\[
\max_{c_{k,n} p_{k,n}} \sum_{k=1}^{K} \sum_{n=1}^{N} c_{k,n} \log_2 \left( 1 + p_{k,n} H_{k,n} \right)
\]

Subject to:

\[
\begin{align*}
C1: & \quad c_{k,n} \in \{0,1\} \ \forall k, n \\
C2: & \quad p_{k,n} \geq 0 \ \forall k, n \\
C3: & \quad \sum_{k=1}^{K} c_{k,n} = 1 \ \forall n \\
C4: & \quad \sum_{k=1}^{K} \sum_{n=1}^{N} c_{k,n} p_{k,n} \leq P_{\text{tot}} \\
C5: & \quad R_i - R_j = \theta_i - \theta_j \ \forall i,j \in \{1, \ldots, K\}, \ i \neq j
\end{align*}
\]

Where \( c_{k,n} \) is the subcarrier allocation indicator such that \( c_{k,n} = 1 \) if and only if subcarrier \( n \) is assigned to user \( k \), and \( P_{\text{tot}} \) is the transmit power constraint. In C5

\[
R_k = \frac{B}{N} \sum_{n=1}^{N} c_{k,n} r_{k,n}
\]

is the total data rate for user \( k \) and \( \theta_1 : \theta_2 : \cdots : \theta_K \) are the normalized proportionality constants where \( \sum_{k=1}^{K} \theta_k = 1 \). The constraints C1 and C2 ensure the correct values for the subcarrier allocation indicator and the power, respectively. C3 imposes the restriction that each subcarrier can only be assigned to one user, and C4 and C5 are the power and proportional rate constraints respectively.

### III. PROPOSED SOLUTIONS

We utilize a combination of the approaches and to exploit the nature of the problem and collect significant complexity reduction benefits while maintaining reasonable performance. The proposed steps are as follows:

1. **Step 1 - Number of subcarriers per user**
   
   In this initial step, we determine \( N_i \) to satisfy
   
   \[
   N_i : N_1 : \cdots : N_K = \theta_1 : \theta_2 : \cdots : \theta_K
   \]
   
   This initial step is based on the proportion of subcarriers assigned to each user is approximately the same as their eventual rates after power allocation, and thus would roughly satisfy the proportionality constraints. This is accomplished by
   
   \[
   N_k = \lfloor \theta_k N \rfloor
   \]
   
   This may lead to \( N^* = N - \sum_{k=1}^{K} N_k \) unallocated subcarriers.

2. **Step 2 - Subcarrier assignment**
   
   This step allocates the per user allotment of subcarriers \( N_i \) and then the remaining \( N^* \) subcarriers
in a way that maximizes the overall capacity while maintaining rough proportionality.

a) Initialization

\[ c_{k,n} = 0, \forall k \in \{1,2,\ldots,K\} \quad \text{and} \quad \forall n \in \{1,2,\ldots,N\} \]

\[ R_k = 0, \forall k \in \{1,2,\ldots,K\} \]

\[ p = \frac{P_{\text{tot}}}{N^2} \]

\[ N = \{1,2,\ldots,N\} \]

b) For \( k = 1 \) to \( K \)

Sort \( H_{k,n} \) in ascending order

\[ n = \arg \max_{n \in N} |H_{k,n}| \]

\[ c_{k,n} = 1 \]

\[ N_k = N_k - 1, N = N \backslash \{n\} \]

\[ R_k = R_k + \frac{p}{N} \log_2(1 + pH_{k,n}) \]

\[ \text{While } \|N\| > 0 \]

\[ \mathcal{K} = \{1,2,\ldots,K\} \]

\[ k = \arg \min_{k \in \mathcal{K}} \frac{R_k}{\phi_k} \]

\[ \text{If } N_k > 0 \]

\[ c_{k,n} = 1 \]

\[ N_k = N_k - 1, N = N \backslash \{n\} \]

\[ R_k = R_k + \frac{p}{N} \log_2(1 + pH_{k,n}) \]

\[ \mathcal{K} = \mathcal{K} \backslash \{k\} \]

c) \[ \mathcal{K} = \{1,2,\ldots,K\} \]

d) \[ \mathcal{K} = \{1,2,\ldots,K\} \]

For \( n = 1 \) to \( N' \)

\[ k = \arg \max_{k \in \mathcal{K}} |H_{k,n}| \]

\[ c_{k,n} = 1 \]

\[ R_k = R_k + \frac{p}{N} \log_2(1 + pH_{k,n}) \]

\[ \mathcal{K} = \mathcal{K} \backslash \{k\} \]

The first step of the algorithm initializes all the variables. \( R_j \) keeps track of the capacity for each user and \( N \) is the set of yet unallocated subcarriers.

The second step assigns to each user the unallocated subcarrier that has the maximum gain for that user. The inherent advantage is gained by the users that are able to choose their best subcarrier earlier than others, particularly for the case of two or more users having the same subcarrier as their best. However, this bias is negligible when \( N \geq K \) since the probability of that happening will be very low.

The third step proceeds to assign subcarriers to each user according to the greedy policy. Since we are enforcing proportional rates, the need of a user is determined by the user who has the least capacity divided by its proportionality constant. Once the user gets his allotment of \( N_i \) subcarriers, he can no longer be assigned any more subcarriers in this step.

The fourth step assigns the remaining \( N' \) subcarriers to the best users for them, wherein each user can get at most one unassigned subcarrier. This is to prevent the user with the best gains to get the rest of the subcarriers. This policy balances achieving proportional fairness while increasing overall capacity. Notice that as a consequence of our subcarrier allocation scheme, \( N_1 : N_2 : \cdots : N_K \approx \varphi_1 : \varphi_2 : \cdots : \varphi_K \)

(7)

with the approximation getting tighter as \( N \to \infty \) and \( N > K \).

C. Step 3 - Power allocation among users

The output of the first two steps is a subcarrier allocation for each user, which reduces the resource allocation problem to an optimal power allocation. We can approximate the relax constraint \( C_3 \) to

\[ R_i; R_j = N_i; N_j; \ \forall i,j \in \{1,\ldots,K\}, \quad i \neq j \]

(8)

Hence, we could replace \( \varphi_i \) with \( N_i \) thus forming simultaneous linear equations which can be written in matrix form as

\[ \begin{bmatrix} 1 & 0 & \cdots & 1 \\ 1 & a_{2,2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 0 & \cdots & a_{k,k} \end{bmatrix} \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_k \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_k \end{bmatrix} \]

(9)

Where

\[ a_{k,k} = \frac{N_k}{N_k} \frac{h_{k,1}w_{k,1}}{h_{1,1}w_{1,1}} \]

(10)

\[ b_k = \frac{N_k}{N_k} \left( W_k - W_1 + \frac{h_{1,1}w_{1,1}}{h_{1,1}w_{1,1}} - \frac{h_{k,1}w_{k,1}}{N_k} \right) \]

(11)

This set of simultaneous linear equations can be easily solved due to its well ordered symmetric and sparse structure.

We first reorder the equations into

\[ \begin{bmatrix} a_{k,k} & 0 & \cdots & 1 \\ 0 & a_{k-1,k-1} & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix} \begin{bmatrix} p_k \\ p_{k-1} \\ \vdots \\ p_1 \end{bmatrix} = \begin{bmatrix} b_K \\ b_{k-1} \\ \vdots \\ b_1 \end{bmatrix} \]

(12)

and then perform LU factorization on the coefficient matrix to obtain

\[ L = \begin{bmatrix} 1 & 0 & \cdots & 0 \\ 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 1 & 1 & \cdots & 1 \end{bmatrix} \]

(13)

\[ U = \begin{bmatrix} a_{k,k} & 0 & \cdots & 1 \\ 0 & a_{k-1,k-1} & \cdots & 1 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 1 - \Sigma_{k=2}^{K} \frac{1}{a_{k,k}} \end{bmatrix} \]

(14)

Using the forward-backwards substitution, the individual powers are given by
\[ P_k = \begin{cases} \left( P_{\text{tot}} - \sum_{k=2}^{K} \frac{b_k}{a_{kk}} \right) / \left( 1 - \sum_{k=2}^{K} \frac{1}{a_{kk}} \right) \quad & \text{for } k = 1 \\ \left( b_k - P_1 \right) / a_{kk} \quad & \text{for } k = 2, \ldots, K \end{cases} \]  \tag{15}

D. Step 4 – Power allocation across subcarriers per user

Step 3 gives the total power \( P_k \) for each user \( k \), which are then used in this final step to perform waterfilling across the subcarrier for each user as

\[ p_{k,n} = p_{k,1} + \frac{h_{k,n} - h_{k,1}}{h_{k,n} h_{k,1}} \]  \tag{16}

\[ p_{k,1} = \frac{p_k - v_k}{n_k} \]  \tag{17}

IV. COMPLEXITY IN ALGORITHM

In order to analyze the computational complexity of the algorithm, recall that \( K \) refers to the total number of users in the system, \( N \) on the other hand refers to the number of subcarriers, which is a power of 2 and much larger than \( K \).

Step 1 of the algorithm requires 1 division and \( K \) multiplications, and thus has a complexity of \( O(K) \).

Step 2(a) requires constant time for initialization. Step 2(b) involves sorting the subcarrier gains \( h_{k,n} \) for each user \( k \), therefore requiring \( O(K N \log_2 N) \) operations. Step 2(c) searches for the best user \( k \) among \( K \) users for the remaining \( N-K \) unallocated subcarriers, thus require \( O\left((N-K) \times K\right) \) operations. Step 2(d) allocates the very few remaining \( N-k \) subcarriers to the best user, and thus requires \( O(K) \) operations. These operations pertain to the subcarrier allocation, and the asymptotic complexity is \( O(KN \log_2 N) \).

Step 3 involves solving for the individual powers. This requires only 1 division, \( 2(K-1) \) multiplications and \( 3(K-1) \) subtractions, thus the complexity is \( O(K) \).

The power allocation step of the \textit{ROOT-FINDING} method, on the other hand, requires iterative root-finding methods such as Newton-Raphson method, bisection method, secant method, and many others. A popular algorithm for solving these equations combines the bisection and secant methods, and is called the ZEROIN subroutine. This is the method used in the simulations, and the complexity is \( O(nK) \), where \( n \) is the number of function evaluations. \( n \) is typically around 10 for smooth functions. Although also asymptotically linear, each function evaluation of involves taking non-integer powers of real numbers. This is significantly more complex than the simple operations required in the \textit{LINEAR} power allocation method, especially when considering implementation in fixed point arithmetic. Furthermore, the \textit{ROOT-FINDING} power allocation method also needs a high subchannel SNR assumption to function properly, which the \textit{LINEAR} method does not make.

Note that Steps 1 and 2 also correspond to the subcarrier allocation step of \textit{ROOT-FINDING}. Both methods involve similar computations and have the same asymptotic complexity. Thus, the real computational savings of \textit{LINEAR} can be seen in the power allocation step.

V. SIMULATION RESULTS

A. Simulation Parameters

The frequency selective multipath channel is modeled as consisting of six independent Rayleigh multipath, with an exponentially decaying profile. A maximum delay spread of 5\,\mu s and maximum doppler of 30\,Hz is assumed. The channel information is sampled every 0.5 ms to update the subchannel and power allocation. The total power was assumed to be 1W, the total bandwidth as 1 MHz, and total subcarriers as 64. The average subchannel SNR is 38 dB, and BER \( \leq 10^{-3} \), giving an SNR gap \( \Gamma = -\ln(5 \times 10^{-3})/1.6 = 3.3 \). This constant is used in the calculation of the rate \( r_{k,n} \) of user \( k \) in subcarrier \( n \) given in (2).

The number of users for the system is varied from 2-16 in increments of 2. A total of 1000 different channel realizations and 100 time samples for each realization were used for each of the number of users. For each channel realization, a set of proportionality constants (expressed as integers) \( \zeta_k \leq \phi_k \) in \( \min \phi_k \) are assigned to each user. It is assumed that these constants follow the probability mass function

\[ p_{\zeta_k} = \begin{cases} 1 \text{ with probability 0.5} \\ 2 \text{ with probability 0.3} \\ 4 \text{ with probability 0.2} \end{cases} \]  \tag{18}

B. Computational Complexity

Fig. 2 shows the computational complexity of the \textit{LINEAR} method. The algorithm compiled from MATLAB code, and was run on a Pentium-4, 3.2 GHz based personal computer running Windows 7 professional. Simulation used floating-point arithmetic. \textit{LINEAR} is an order of magnitude faster in execution time than \textit{ROOT-FINDING}.

C. Overall Capacity

Fig. 3 shows the total capacities of the proposed \textit{LINEAR} method. The capacities increase as the number of users increases. This is the effect of multiuser diversity gain, which is more prominent in systems with larger number of users. The proposed \textit{LINEAR} method has a consistently higher total capacity than the \textit{ROOT-FINDING} method for all the numbers of users for this set of simulation parameters.
CONCLUSION

This paper presents a new method to solve the rate-adaptive resource allocation problem with proportional rate constraints for OFDMA systems. It improves on the previous work in this area [9] by developing a novel subcarrier allocation scheme that achieves approximate rate proportionality while maximizing the total capacity. This scheme was also able to exploit the special linear case in [9], thus allowing the optimal power allocation to be performed using a direct algorithm with a much lower complexity versus an iterative algorithm. It is shown through simulation that the proposed method performs better than the previous work in terms of significantly decreasing the computational complexity, and yet achieving higher total capacities, while being applicable to a more general class of systems.

REFERENCES