International Journal of Industrial Electronics and Electrical Engineering, ISSN(p): 2347-6982, ISSN(e): 2349-204X
Volume-6, Issue-7, Jul.-2018, http://ijieee.org.in

DECENTRALIZED CONTROL OF A MULTIVARIABLE SYSTEM

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Abstract - A complex multi-axis machinery can be represented as a combination of local subsystems with interconnections between channels and couplings through the machine structure or process materials. Interactions between subsystems can complicate a control problem, especially when the parameters of the plant are of time-varying or uncertain. The adaptive control system described in this paper allows one to achieve an independent motion of plant outputs due to the elimination of interconnections between channels by local feedbacks, and simultaneously compensates the uncertainties of the plant parameters. The desirable motion of the subsystems outputs can be accomplished by using relevant reference models in each local subsystem.

Index terms - Multivariable system, time-varying systems, feedback control, system stability.

I. TASK FORMULATION

The method described below is considered to realize the practical task – speed control of a number of electric motors in an industrial conveyer with a variable load. Motors with similar parameters are connected in clusters. Each cluster represents a subsystem with its own controller and a reference model. It is suggested in this paper that the control system is designed in such a way that the followings will be achieved:

- local subsystems are decoupled from interconnections;
- each subsystem follows the desirable trajectory of a reference model.

A multivariable plant can be described by the following matrix differential equations:

$$\dot{x}_1 = A_{11}x_1 + A_{12}x_2 + ... + A_{1n}x_n + u_1,$$
$$\dot{x}_2 = A_{21}x_1 + A_{22}x_2 + ... + A_{2n}x_n + u_2,$$
$$\cdots\cdots\cdots$$
$$\dot{x}_n = A_{n1}x_1 + A_{n2}x_2 + ... + A_{nn}x_n + u_n, \quad (1)$$

where

$$x_1 \in R^{\ell_1},\ x_2 \in R^{\ell_2},\ ....,\ x_n \in R^{\ell_n},$$
$$u_1 \in R^{\ell_1},\ u_2 \in R^{\ell_2},\ ....,\ u_n \in R^{\ell_n},$$

$x_1, x_2, ..., x_n$ - are state vectors of subsystems,
$A_{11} = A_{11}(t),\ A_{12} = A_{12}(t), ..., A_{nn} = A_{nn}(t)$ - are time-variable matrices of parameters,
$u_1, u_2, ..., u_n$ - are control vectors.
$\ell_1 = \ell_{11}\ell_{12}, \ell_2 = \ell_{21}\ell_{22}, ..., \ell_n = \ell_{n1}\ell_{n2}$
ℓ_{i1} - is the order of a model of an electrical motor in the i-th cluster, $(i = 1,2,...,n)$
ℓ_{i2} - is the number of electrical motors in the i-th cluster.

It is required to find such feedback laws that independent motion of local subsystems follows the reference models:

$$\dot{x}_1^m = A_{11}^m x_1^m + g_1,$$
$$\dot{x}_2^m = A_{22}^m x_2^m + g_2, \quad (2)$$
$$\cdots\cdots\cdots$$
$$\dot{x}_n^m = A_{nn}^m x_n^m + g_n,$$

where

$x_1^m, x_2^m, ..., x_n^m$ - are state vectors of reference models,
$A_{11}^m,\ A_{22}^m, ..., A_{nn}^m$ - are matrices of parameters of reference models,
$g_1, g_2, ..., g_n$ - are command vectors.

II. SYNTHESIS OF THE BASIC LOOP'S STRUCTURE

The feedback controls can be chosen as follows:

$$u_1 = g_1 + K_{11}\hat{x}_1 + K_{12}\hat{x}_2 + ... + K_{1n}\hat{x}_n,$$
$$u_2 = g_2 + K_{21}\hat{x}_1 + K_{22}\hat{x}_2 + ... + K_{2n}\hat{x}_n, \quad (3)$$
$$\cdots\cdots\cdots$$
$$u_n = g_n + K_{n1}\hat{x}_1 + K_{n2}\hat{x}_2 + ... + K_{nn}\hat{x}_n,$$

where

$K_{11} = K_{11}(t), K_{12} = K_{12}(t), ..., K_{nn} = K_{nn}(t)$ are matrices of feedback gains,
$\hat{x}_1, \hat{x}_2, ..., \hat{x}_n$ - are state estimates of subsystems.

According to the equations (1) and (3), the closed loop system can be represented as follows:

$$\begin{aligned}\dot{\hat{x}}_i &= (A_{i1}+K_{i1})\hat{x}_1 + (A_{i2}+K_{i2})\hat{x}_2 + \\ &\ldots + (A_{in}+K_{in})\hat{x}_i + g_i, \quad (i=1,2,\ldots,n).\end{aligned} \quad (4)$$

It can be seen from equation (4) that the system "plant + controller" will be dynamically decoupled on each channel if the following conditions are achieved:

$$A_{ij}+K_{ij}=0, \qquad (i,j=1,2,\ldots n, \quad i\neq j). \quad (5)$$

In order to obtain the desirable motion of the system according to the reference model (2), the following relation should be provided:

$$A_{ii}+K_{ii}=A_{ii}^{m}\ . \quad (6)$$

III. SYNTHESIS OF ADAPTIVE CONTROL ALGORITHMS

The variable parameters of the plant (1) can be represented as follows:

$$A_{ij}=A_{ij}^{0}+\Delta A_{ij},$$

where

A_{ij}^{0} =const., (i,j=1,2,…,n), A_{ij}^{0} - are constant matrices whose coefficients correspond to the nominal operation mode of the subsystem,
$\Delta A_{ij}^{0}=\Delta A_{ij}^{0}(t)$, ΔA_{ij}^{0} - are deviations of plant parameters from the nominal operation mode.

The controller parameters can be represented analogously:

$$K_{ij}=K_{ij}^{0}+\Delta K_{ij},$$

where

K_{ij}^{0} =const., (i,j=1,2,…,n), K_{ij}^{0} - are matrices of constant coefficients of the controller,
$\Delta K_{ij}=\Delta K_{ij}(t)$, ΔK_{ij} - are matrices of adjustable coefficients of the controller.

Thus, the motion of the subsystems' states on a non-nominal operation mode can be represented as follows:

$$\dot{x}_i=\sum_{j=1}^{n}(A_{ij}^{0}+\Delta A_{ij}(t))x_j+u_i \ \text{(i=1,2,…,n).} \quad (7)$$

The feedback control is:

$$u_i=g_i+\sum_{j=1}^{n}(K_{ij}^{0}+\Delta K_{ij}(t))x_j. \quad (8)$$

Substituting equation (8) in (7) one can obtain the equation of motion of the system "plant + controller" on each channel:

$$\begin{aligned}\dot{x}_i &= \sum_{j=1}^{n}(A_{ij}^{0}+\Delta A_{ij}(t))x_j + \\ &\sum_{j=1}^{n}(K_{ij}^{0}+\Delta K_{ij}(t))x_j+g_i.\end{aligned} \quad (9)$$

The model motion equation on each channel can be chosen as follows:

$$\dot{x}_i^{m}=A_{ii}^{m}x_i^{m}+\sum_{\substack{j=1\\ j\neq i}}^{n}(A_{ij}^{0}+K_{ij}^{0})x_j+g_i. \quad (10)$$

Comparing equations (9) and (10) and taking into account that $A_{ii}^{0}+K_{ii}^{0}=A_{ii}^{m}$, we can obtain error motion equation:

$$\begin{aligned}e_i &= A_{ii}^{m}e_i+(\Delta A_{ii}(t)+\Delta K_{ii}(t))x_i + \\ &(\sum_{\substack{j=1\\ j\neq i}}^{n}\Delta A_{ij}(t)+\sum_{\substack{j=1\\ j\neq i}}^{n}\Delta K_{ij}(t))x_j,\end{aligned} \quad (11)$$

where

$$e_i=x_i-x_i^{m}\ . \quad (12)$$

Let's introduce the following notation:

$$Y_{ij}=Y_{ij}(t)=\Delta A_{ij}(t)+\Delta K_{ij}(t)\ . \quad (13)$$

Therefore, the error equation (11) can be represented as:

$$\dot{e}_i=A_{ii}^{m}e_i+Y_{ii}x_i+\sum_{\substack{j=1\\ j\neq i}}^{n}Y_{ij}x_j\ . \quad (14)$$

The adaptive control algorithms can be specified in the following form:

$$\frac{d}{dt}\Delta K=\Psi\ . \quad (15)$$

(Here and further we omit indexes i, j for simplicity, unless it will be required).

Given that parameters' deviation $\Delta A = \Delta A(t)$ is differentiable in time:

$$\frac{d}{dt}\Delta A(t) = R(t)\,. \qquad (16)$$

Therefore, according to equations (12) – (16) the following can be obtained:

$$\begin{aligned} \dot{e} &= A^m e + Ye, \\ Y &= \Psi + R. \end{aligned} \qquad (17)$$

where $R = R(t)$.

The equations (17) and (2) describe the dynamic motion of the adaptive control system with the reference model.

Given that the matrix Ψ is a function of error deviation e and time t, and $\Psi(e,t) = 0$ at t = 0.

In this case the state and parameters error motions

$$e \equiv 0, \quad Y \equiv 0\,, \qquad (18)$$

with $R(t) \equiv 0$ (according to the hypothesis of quasi-stationary) is the solution of the system (17).

It is suggested to use the second method of Lyapunov in order to obtain the adaptive control algorithms from the condition of zero stability solution (18) of the system (17) [1] , [2], [3] and [4].

The quadratic function can be chosen as follows:

$$V = \gamma e^T P e + tr(YY^T)\,, \qquad (19)$$

where

γ -const., P - is a symmetric matrix.

The time derivative for the function V is obtained as follows:

$$\begin{aligned} &\dot{V} = \gamma \dot{e}^T P e + \gamma e^T P \dot{e} + tr(\dot{Y}Y^T + Y\dot{Y}^T) = \\ &\gamma(A^m e + Yx)^T P e + \gamma e^T P(A^m e + Yx) + \\ &tr(\dot{Y}Y^T + Y\dot{Y}^T) = \gamma[(A^m e)^T + (Yx)^T]Pe + \\ &\gamma e^T P(A^m e + Yx) + tr(\dot{Y}Y^T + Y\dot{Y}^T) = \\ &\gamma[e^T (A^m)^T + x^T Y^T]Pe + \gamma e^T PA^m e + \\ &\gamma e^T PYx + tr(\dot{Y}Y^T + Y\dot{Y}^T) = \\ &\gamma e^T (A^m)^T Pe + \gamma x^T Y^T Pe + \gamma e^T PA^m e + \\ &\gamma e^T PYx + tr(\dot{Y}Y^T + Y\dot{Y}^T) = \end{aligned}$$

$$\begin{aligned} &\gamma e^T[(A^m)^T P + PA^m]e + \gamma x^T Y^T Pe + \\ &\gamma e^T PYx + tr(\dot{Y}Y^T + Y\dot{Y}^T). \end{aligned} \qquad (20)$$

Following some intermediate matrix manipulations we obtain:

$$\Psi = -\gamma Pex^T\,. \qquad (21)$$

The derivative of the Lyapunov function is represented as:

$$\dot{V} = \gamma e^T Q e\,, \qquad (22)$$

where $Q = (A^m)^T P + PA^m$, Q - is the negative definite matrix.

Taking into account that A^T is a Hurwitch matrix, the following is obtained:

$$\dot{V} \le 0\,. \qquad (23)$$

Therefore, according to equations (17), (15) and using conditions of quasi-stationary of the system, one can obtain the adaptive control algorithms as:

$$\Delta\dot{K} = -\gamma Pex^T \qquad (24)$$

It follows from the above that motion of the system (17) with the algorithms (24) is stable and conditions (5) and (6) are satisfied.

CONCLUSUONS

The adaptive algorithms described in this paper allow one to achieve independent control of separate subsystems with uncertain parameters in a complex multi-axis machinery. The desirable behavior of each sub-system can be obtained according to a reference model of the relevant sub-system. The stability of the overall system is guaranteed according to the stability criteria based on the Lyapunov function.

RFERENCES

[1] Astrom K.J. and Wittenmark B., “Adaptive Control”, Mass.; Addison – Wesley, 1995.

[2] Vershinin Y. A., “Guarantee of tuning independence in multivariable systems”, Proceedings The Fifth Leningrad Conference on theory of adaptive control systems: “Adaptive and expert systems in control”, Leningrad, 1991.

[3] Landau I.D., Lozano R., M'Saad M. and Karimi A., “Adaptive Control. Algorithms, analysis and applications”, Springer, 2011. .

[4] Kumpati S. Narendra and Anuradha M. Annaswamy, “Stable adaptive systems”, Publisher Courier Corporation, 2012.

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