INTERVAL TYPE-2 FUZZY REPLACEMENT ANALYSIS IN CALL CENTER INVESTMENT REPLACEMENT DECISIONS

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Abstract— Professionals need to make replacement decisions if current asset can no longer meet actual needs, if a new technology occurs or if current asset has a fatal breakdown. Inevitably, replacement decisions are mostly made in an uncertain financial investment environment. In this paper, in order to cover this uncertainity and to get through the critics of ordinary fuzzy sets, a type-2 fuzzy replacement analysis technique is developed using trapezoidal interval type-2 fuzzy sets. The method developed is also applied to a numerical application of a replacement problem. A genralized graphical representation of trapezoidal interval type-2 fuzzy sets also presented for ease of decision makers. The result has came up explicit and considerable outputs showing that the technique developed can be used in replacement challenges.

Keywords— Internal type-2 fuzzy sets, fuzzy replacement analysis, defuzzification, sensitivity analysis.

I. INTRODUCTİON

Higher technology based products and new services are becoming more widely used by cooperations and individuals. New product development, ingenious innovative marketing channels and cost effective production systems make products and services more reacheable for both firms and end-users. Thus, most decisions made involve replacement decision making whether to acquire a new model of an existing product or service.If we evaluate these kind of renewal or refurbishment decisions from a replacement analysis stand point of engineering economy, we would easily perceive and internalize the importance of replacement analysis and its theory.

We notice that one of the most current problem of replacement analysis is that the motivation behind the decisions have been changed dramatically. Eventough the main reasons such as the insufficiency of existing asset, low cost of a newly developed challenger, or the detoriation of existing asset; are still keeping their importance,they are not the only reasons anymore. Decision makers could change an existing asset even if it does meet the current needs, or an existing asset could be replaced even if it is running perfectly and has no defect at all.

We believe that these kind of decision reasons have not only economic and financial aspects but also sociologic and physchologic motivations. Whatever the reasons behind are, the remarkable change in renewal compulsions exhibits the growing importance of replacement analysis. Thus, new methods, approaches and frameworks in decision making space of replacement analysis should be developed in order to face and meet the upcoming regeneration wave.

The disposition of the paper is as follows: In the next section we will present the earlier efforts conducted in both replacement analysis and fuzzy based capital budgeting techniques and approaches as well as type-2 fuzzy sets. In the third section we will share the main structure of fuzzy mathematics to be used in the paper. The fourth section would cover the interval type-2 fuzzy replacement analysis framework development. The defuzzification will be presented in the fifth section and the sixth section will discuss a numerical application and its graphical representation following by a sensitivity analysis section and the conclusion.

II. LITERATURE REVIEW: FUZZY ENGINEERING ECONOMY METHODS

First replacement studies have been conducted in 1950's. In 1955, Bellmann [2] studied replacement with dynamic programming. Dreyfus [13] added the repair decision approach in 1960. In 1972, Dreyfus [14] presented the dynamic replacement model, and in 1979 Lake et al. [30] presented the equipment replacement models. In 1984, Oakford [36] has developed examples on these models presented by Lake at al. Lohmann [33] has progressed this work with a sthocastic approach in 1986.

After Zadeh [46] has introduced the fuzzy concepts, and after Bellmann et al. [3] has developed fuzzy approach to decision making problems, many researchers started to conduct studies on fuzzy decision making and fuzzy applications on finance. In 1987, Buckley [5] has introduced the foundations of fuzzy finance and in 1992 he has exhibits fuzzy financial equations [6].Chiu et al. [9] developed fuzzy cash flow analysis in 1994 and in 1998, they studied capital budgeting decisions on fuzzy projects [10].Karsak [26], Boussabaine [4] and Kahraman et al. [22] have made different studies on liquidity risk, cash flow analysis and benefit/cost ratio analysis with fuzzy sets, respectively.Kutcha [28] and Kahraman [18] [19] progressed the earlier works on fuzzy capital budgeting. Karsak et al. [27] has developed a fuzzy framework for manufacturing investments in 2001. In 2002, in 2003 and in 2004, Kahraman et al. [20] [21] [23] conducted fuzzy studies on capital budgeting techniques using discounted fuzzy versus probabilistic cash flows, dynamic programming and flebility of manufacturing systems using fuzzy cash flows, respectively.

Usher [44] developed first fuzzy examples of life cycle cost in 1993. Esogbue et al. [15] has developed first fuzzy replacement approaches in 1998.

With the leading studies of Karnik et al. [25]and Mendel et al. [34], researchers have started to focus on interval type-2 fuzzy sets. In 2008 and in 2010, Lee et al. [31] and Chen et al. [8] studied interval type-2 fuzzy sets with TOPSIS method and presented numerical examples. Co et al. [11], Kuo-Ping [29], Gong [16] and Aras et al. [1] have also conducted interval type-2 fuzzy studies on control systems and multiple criteria decision making.

In 2005, Yao et al. [45] studied valuation using a fuzzy discounted cash flow model. In 2005, Tolga et al. [42] ve Chang [7] studied on fuzzy replacement models. In 2006, Kahraman et al. [24] has discussed applications of fuzzy capital budgeting techniques and Sevastijanov et al. [39] presented investment project evaluation and optimization using fuzzy capital budgeting techniques in Kahraman's book. Liou et al. [32] conducted a reasearch on fuzzy decision making using fuzzy annual worth for alternative selection.In 2007, Huang [17] and Omitaomu et al. [37] conducted fuzzy present value analysis in their papers. In 2008, Sevastjanovet al. [40] discussed on the fuzzy internal rate of return in Kahraman's book. In 2008, Demircan et al. [12] has presented the generalized fuzzy replacement framework. In 2011 and in 2012, Shahriari [41] and Tsao [43] studied mapping fuzzy approach in engineering economy and fuzzy net present value for capital investments, respectively. In 2015, Sari at al. [38] conducted interval type-2 fuzzy capital budgeting studies.

III. INTERVAL TYPE-2 FUZZY SETS

All the main mathematical operations of interval type-2 fuzzy sets will be calculated based on Mendel et al. [34] and Lee et al. [31], studies and all fuzzy sets used will be trapezoidal interval type-2 fuzzy sets as shown in Fig.1.

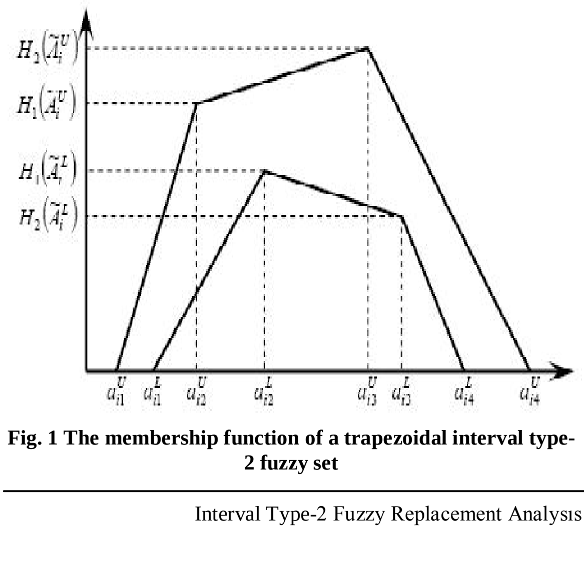

Fig. 1 The membership function of a trapezoidal interval type-2 fuzzy set

$$\tilde{\tilde{A}}_i = \left(\tilde{A}_i^U, \tilde{A}_i^L\right) = ((a_{i1}^U, a_{i2}^U, a_{i3}^U, a_{i4}^U; H_1(\tilde{A}_i^U), H_2(\tilde{A}_i^U)), (a_{i1}^L, a_{i2}^L, a_{i3}^L, a_{i4}^L; H_1(\tilde{A}_i^L), H_2(\tilde{A}_i^L))) \quad (1)$$

Mathematical operations over trapezoidal interval type-2 fuzzy sets will be defined as follows; for addition operation;

$$\begin{aligned}\tilde{\tilde{A}}_1 \oplus \tilde{\tilde{A}}_2 &= \left(\tilde{A}_1^U, \tilde{A}_1^L\right) \oplus \left(\tilde{A}_2^U, \tilde{A}_2^L\right)\\ &= ((a_{11}^U + a_{21}^U, a_{12}^U + a_{22}^U, a_{13}^U + a_{23}^U, a_{14}^U + a_{24}^U;\\ &\min(h_{11}^U, h_{21}^U), \min(h_{12}^U, h_{22}^U)),\\ &(a_{11}^L + a_{21}^L, a_{12}^L + a_{22}^L, a_{13}^L + a_{23}^L, a_{14}^L + a_{24}^L;\\ &\min(h_{11}^L, h_{21}^L), \min(h_{12}^L, h_{22}^L)))\end{aligned} \quad (2)$$

for substraction operation;

$$\begin{aligned}\tilde{\tilde{A}}_1 \ominus \tilde{\tilde{A}}_2 &= \left(\tilde{A}_1^U, \tilde{A}_1^L\right) \ominus \left(\tilde{A}_2^U, \tilde{A}_2^L\right)\\ &= ((a_{11}^U - a_{21}^U, a_{12}^U - a_{22}^U, a_{13}^U - a_{23}^U, a_{14}^U - a_{24}^U;\\ &\min(h_{11}^U, h_{21}^U), \min(h_{12}^U, h_{22}^U)),\\ &(a_{11}^L - a_{21}^L, a_{12}^L - a_{22}^L, a_{13}^L - a_{23}^L, a_{14}^L - a_{24}^L;\\ &\min(h_{11}^L, h_{21}^L), \min(h_{12}^L, h_{22}^L)))\end{aligned} \quad (3)$$

for multiplication operation;

$$\begin{aligned}\tilde{\tilde{A}}_1 \otimes \tilde{\tilde{A}}_2 &= \left(\tilde{A}_1^U, \tilde{A}_1^L\right) \otimes \left(\tilde{A}_2^U, \tilde{A}_2^L\right)\\ &= ((a_{11}^U \times a_{21}^U, a_{12}^U \times a_{22}^U, a_{13}^U \times a_{23}^U, a_{14}^U \times a_{24}^U;\\ &\min(h_{11}^U, h_{21}^U), \min(h_{12}^U, h_{22}^U)),\\ &(a_{11}^L \times a_{21}^L, a_{12}^L \times a_{22}^L, a_{13}^L \times a_{23}^L, a_{14}^L \times a_{24}^L;\\ &\min(h_{11}^L, h_{21}^L), \min(h_{12}^L, h_{22}^L)))\end{aligned} \quad (4)$$

and for division operation;

$$\begin{aligned}\tilde{\tilde{A}}_1 \odot \tilde{\tilde{A}}_2 &= \left(\tilde{A}_1^U, \tilde{A}_1^L\right) \odot \left(\tilde{A}_2^U, \tilde{A}_2^L\right)\\ &= ((a_{11}^U \times a_{24}^U, a_{12}^U \times a_{23}^U, a_{13}^U \times a_{22}^U, a_{14}^U \times a_{21}^U;\\ &\min(h_{11}^U, h_{21}^U), \min(h_{12}^U, h_{22}^U)),\\ &(a_{11}^L \times a_{24}^L, a_{12}^L \times a_{23}^L, a_{13}^L \times a_{22}^L, a_{14}^L \times a_{21}^L;\\ &\min(h_{11}^L, h_{21}^L), \min(h_{12}^L, h_{22}^L)))\end{aligned} \quad (5)$$

IV. TYPE-2 FUZZY REPLACEMENT ANALYSİS

The developed fuzzy replacement formula by Tolga et al. [42]will be used while$\tilde{\tilde{I}}$representsthe type-2 fuzzy initial investment, $\tilde{\tilde{S}}_T$represents the type-2 fuzzy salvage value at period T and finally,$\tilde{\tilde{C}}_T$represents type-2 fuzzy capital recovery (6), where $\tilde{\tilde{R}}$stands for type-2 fuzzy discount rate. Please note that the type-2 fuzzy initial investment $\tilde{\tilde{I}}$would be considered as crisp in the numerical application and in the sensitivity analysis, since it does not contain any fuzziness in real life examples.

$$\tilde{\tilde{C}}_T = \left(\tilde{C}_T^U, \tilde{C}_T^L\right) = \left(\tilde{\tilde{I}} \ominus \tilde{\tilde{S}}_T\right) \otimes \left(\frac{\tilde{\tilde{A}}}{\tilde{\tilde{P}}}, \tilde{\tilde{R}}, T\right) \oplus \left(\tilde{\tilde{R}} \otimes \tilde{\tilde{S}}_T\right) \quad (6)$$

$$\text{where } \left(\tilde{\tilde{A}}/\tilde{\tilde{P}}, \tilde{\tilde{R}}, T\right) = \frac{\tilde{\tilde{R}} \otimes (1 \oplus \tilde{\tilde{R}})^t}{(1 \oplus \tilde{\tilde{R}})^t \ominus 1} \quad (7)$$

Let $\tilde{\tilde{F}}_T$bethe type-2 fuzzy operating and maintenance cost for period T. Where $\sum \tilde{\tilde{P}}_T$ represents cumulative type-2 fuzzy present value of $\tilde{\tilde{F}}_T$'s, from $t = 1$ to $t = T$, and $\tilde{\tilde{A}}_T$ stands for type-2 fuzzy equivalent uniform annual cost (EUAC) of cumulative type-2 fuzzy present value $\tilde{\tilde{F}}_T$'s over T period(s). As a result the total EUAC, $\tilde{\tilde{W}}_T$ is calculated as;

$$\tilde{\tilde{W}}_T = \tilde{\tilde{C}}_T \oplus \tilde{\tilde{A}}_T = \tilde{\tilde{C}}_T \oplus \sum \tilde{\tilde{P}}_T \otimes \left(\tilde{\tilde{A}}/\tilde{\tilde{P}}, \tilde{\tilde{R}}, T\right) \quad (8)$$

$$\text{where} \tilde{\tilde{P}}_T = \frac{\tilde{\tilde{F}}_t}{(1 \oplus \tilde{\tilde{R}})^t} \quad (9)$$

$$\tilde{\tilde{W}}_T = \left(\tilde{W}_T^U, \tilde{W}_T^L\right) = \tilde{\tilde{C}}_T \oplus \sum_{t=1}^{T} \frac{\tilde{\tilde{F}}_t}{\left(1 \oplus \tilde{\tilde{R}}\right)^t} \otimes \frac{\tilde{\tilde{R}} \otimes \left(1 \oplus \tilde{\tilde{R}}\right)^t}{\left(1 \oplus \tilde{\tilde{R}}\right)^t \ominus 1} \quad (10)$$

The general graphical representation of capital recovery costs $\tilde{\tilde{C}}_T$, equivalent uniform annual operational costs $\tilde{\tilde{A}}_T$ and the total equivalent uniform annual costs $\tilde{\tilde{W}}_T$ are expected to be as drawn in Fig.2.

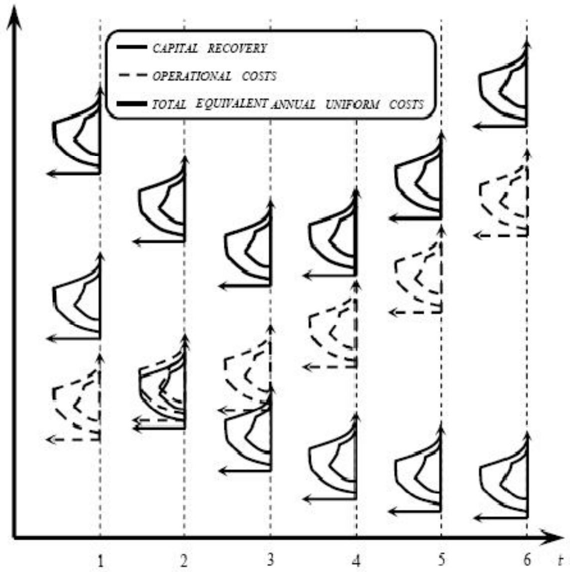

Fig 2.General graphical represenation of capital recovery, operational and total equivalent uniform annual costs

V. DEFUZZIFICATION OF INTERNAL TYPE-2 FUZZY SETS

For the defuzzification of internal type-2 fuzzy sets, we will use the type reduction methodology proposed by Niewiadomski et al. [35] and applied by Sari et al. [38].

First, internal type-2 fuzzy results will be transformed to ordinary type-1 fuzzy sets and then the mode of the transformed type-1 fuzzy set will be calculated and considered as the crisp equivalent.As Niewiadomski et al. [35] proposed; optimistic, pessimistic, realistic and weighted average indices will be determined as follows;

$$TR_{op}\left(\tilde{\tilde{A}}\right) = \tilde{A}_i^U \quad (11)$$

$$TR_{pe}\left(\tilde{\tilde{A}}\right) = \tilde{A}_i^L \quad (12)$$

$$TR_{re}\left(\tilde{\tilde{A}}\right) = \frac{\tilde{A}_i^U + \tilde{A}_i^L}{2} \quad (13)$$

$$TR_{wa}\left(\tilde{\tilde{A}}\right) = w_1 \tilde{A}_i^U + w_2 \tilde{A}_i^L, w_1 + w_2 = 1 \quad (14)$$

The mode m_i,which will be used as a crisp representative of type-1 fuzzy sets, divides the membership function into two equal areas. The mode m_i should be calculated for both $h_{i1} > h_{i2}$(see Fig. 3) and $h_{i1} < h_{i2}$(see Fig. 4) cases;

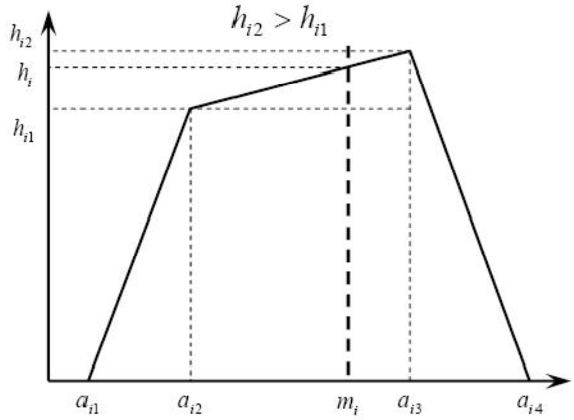

Fig. 3 Graphical represention of median of a trapezoidal fuzzy set while $h_{i1} < h_{i2}$.

for $h_{i1} > h_{i2}, \frac{a_{i3}-m_i}{h_m-h_{i2}} = \frac{a_{i3}-a_{i2}}{h_{i1}-h_{i2}}$, and (15)

$$(a_{i2} + a_{i1}) \times h_{i1} + (h_{i1} + h_m) \times (m_i - a_{i2}) = (a_{i4} - a_{i3}) \times h_{i2} + (h_{i2} + h_m) \times (a_3 - m_i) \quad (16)$$

Using the equations (15) and (16) above, we can easily turn to a second degree equation to find the mode m_i, where;

$X \times m_i^2 + Y \times m_i + Z = 0$, where; (17)

$$X = 2 \times \frac{h_{i1} - h_{i2}}{(a_{i3} - a_{i2})} \quad (18)$$

$$Y = -3 \times h_{i2} - h_{i1} - \frac{h_{i1} - h_{i2}}{(a_{i3} - a_{i2})} \times (3 \times a_{i3} + a_{i2}) \quad (19)$$

$$Z = h_{i1} \times a_{i1} + h_{i2} \times (a_{i2} + a_{i3} + a_{i4}) + \frac{a_{i3} \times (h_{i1} - h_{i2}) \times (a_{i2} + a_{i3})}{(a_{i3} - a_{i2})} \quad (20)$$

for $h_{i1} < h_{i2}, \frac{m_i-a_{i2}}{h_i-H_1(\tilde{A})} = \frac{a_{i3}-a_{i2}}{h_{i2}-h_{i1}}$ (21)

$$(a_{i2} - a_{i1}) \times h_{i1} + (h_{i1} + h_m) \times (m_i - a_{i2}) = (a_{i4} - a_{i3}) \times h_{i2} + (h_{i2} + h_m) \times (a_3 - m_i) \quad (22)$$

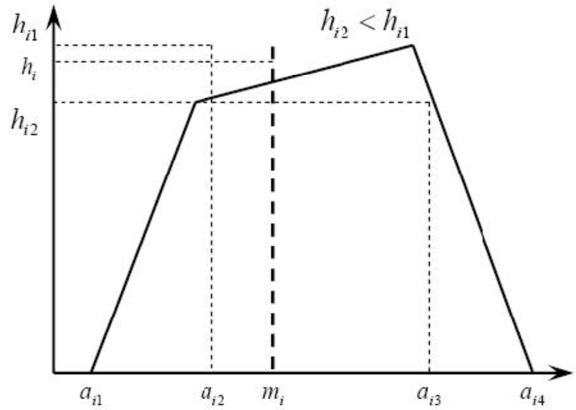

Fig 4.Graphical represention of median of a trapezoidal fuzzy set while $h_{i1} > h_{i2}$.

In this case, the second degree equation to calculate the mode m_i is;

$X \times m_i^2 + Y \times m_i + Z = 0$, where; (23)

$$X = 2 \times \frac{h_{i2} - h_{i1}}{(a_{i3} - a_{i2})} \quad (24)$$

$$Y = 3 \times h_{i1} + h_{i2} - \frac{h_{i2} - h_{i1}}{(a_{i3} - a_{i2})} \times (3 \times a_{i2} + a_{i3}) \quad (25)$$

$$Z = -h_{i2} \times a_{i4} - h_{i1} \times (a_{i1} + a_{i2} + a_{i3}) + \frac{a_{i2} \times (h_{i2} - h_{i1}) \times (a_{i2} + a_{i3})}{(a_{i3} - a_{i2})} \quad (26)$$

VI. NUMERICAL EXAMPLE

Let us assume that we have an equipment with an initial investment of $\tilde{\tilde{I}}$, with its corresponding salvage values $\tilde{\tilde{S}}_T$(see Table 1) over 6 periods, and its related operational costs $\tilde{\tilde{F}}_T$(see Table 2) for each ending-year. Since the initial invesment $\tilde{\tilde{I}}$does not include fuzziness, it will be considered as a discrete financial figure.Salvage values $\tilde{\tilde{S}}_T$are displayed in Table 1, where $\tilde{\tilde{S}}_0 = \tilde{\tilde{I}}$. Discount rate $\tilde{\tilde{R}}$ will also be considered as a type-2 fuzzy set with the following figures; (0.04, 0.06, 0.12, 0.14; 0.90, 1.00)(0.06, 0.08, 0.10, 0.11; 0.70, 0.60).

Table 1 Trapezoidal Interval Type-2 Salvage Values

t	Salvage Value
0	(15000, 15000, 15000, 15000; 1.00, 1.00) (15000, 15000, 15000, 15000; 1.00, 1.00)
1	(3000, 5000, 11000, 12000; 0.70, 0.90) (4000, 7000, 9000, 11000; 0.65, 0.60)
2	(1000, 3000, 6000, 9000; 0.90, 0.80) (2000, 4000, 5000, 8000; 0.65, 0.70)
3	(500, 1000, 4000, 5500; 0.80, 0.90) (500, 700, 1100, 1500; 0.90, 0.80)
4	(500, 700, 1100, 1500; 0.90, 0.80) (600, 800, 1200, 1300; 0.55, 0.60)
5	(0, 0, 0, 0; 1.00, 1.00)(0, 0, 0, 0; 1.00, 1.00)
6	(0, 0, 0, 0; 1.00, 1.00)(0, 0, 0, 0; 1.00, 1.00)

Table 2 Trapezoidal Interval Type-2 Operational Costs

t	Operational Costs
1	(3000, 4000, 6000, 7500; 0.70, 0.90) (3500, 4500, 5500, 6000; 0.60, 0.50)
2	(4400, 4800, 6000, 6100; 0.90, 0.80) (4600, 5000, 5600, 6000; 0.65, 0.70)
3	(5500, 5700, 6700, 7200; 0.85, 0.90) (5700, 6000, 6500, 7000; 0.70, 0.65)
4	(5500, 6500, 9000, 9500; 0.90, 0.80) (6000, 7500, 8500, 8600; 0.70, 0.70)
5	(8200, 8500, 10000, 10300; 0.80, 0.90) (8400, 9000, 10000, 10200; 0.75, 0.65)
6	(9700, 11000, 13200, 13500; 0.90, 0.80) (10000, 12000, 13000, 13200; 0.70, 0.60)

In order to calculate capital recovery costs we are going to use the interval type-2 fuzzy replacement equations (6) and (7), to calculate equivalent uniform annual operating costs we are going to use equations (8) and (9), and finally to calculate the total equivalent uniform annual costs we are going to use the equation (10), respectively.The calculated results and graphical representations are shown in Fig.5 and in Table 3 for capital recovery costs, in Fig.6 and in Table 4 for equivalent uniform annual operational costs and in Fig.7 and in Table 5 for total equivalent uniform annual costs.

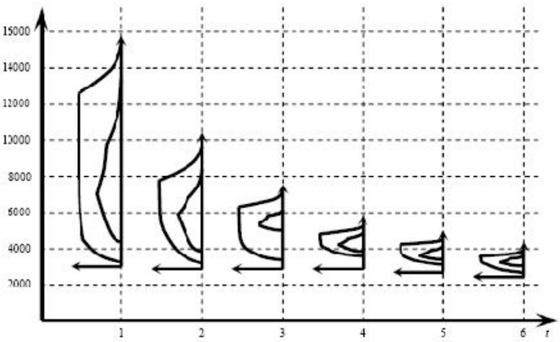

Fig 5.Trapezoidal Internal Type-2 Fuzzy Representation of Capital Recovery Costs

Table 3 Trapezoidal Interval Type-2 Capital Recovery Costs

t	Capital Recovery Costs
1	(3240, 4540, 12520, 15360; 0.70, 0.90) (4480, 7040, 9700, 13420; 0.65, 0.60)
2	(3221.18, 5088.93, 7820.38, 9762.06; 0.90, 0.80) (3938.06, 5927.69, 6838.10, 8471.14; 0.65, 0.60)
3	(3443.31, 4175.21, 6308.89, 7015.61; 0.80, 0.90) (5080.48, 5449.67, 5860.24, 6098.59; 0.70, 0.60)
4	(3739.12, 4053.42, 4840.05, 5186.47; 0.90, 0.80) (3989.70, 4230.51, 4599.69, 4784.50; 0.55, 0.60)
5	(3369.41, 3560.95, 4161.15, 4369.25; 0.90, 1.00)(3560.95, 3756.85, 3956.96, 4058.55; 0.70, 0.60)
6	(2861.43, 3050.44, 3648.39, 3857.36; 0.90, 1.00)(3050.44, 3244.73, 3444.11, 3545.65; 0.70, 0.60)

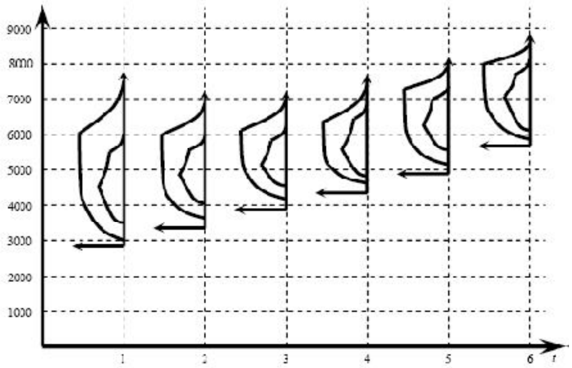

Fig 6.Trapezoidal Internal Type-2 Fuzzy Representation of Equivalent Uniform Annual Operational Costs

Table 4 Trapezoidal Interval Type-2 Fuzzy Equivalent Uniform Annual Operating Costs

t	Equivalent Uniform Annual Operating Costs
1	(3000, 4000, 6000, 7500; 0.70, 0.90) (3500, 4500, 5500, 6000; 0.60, 0.50)
2	(3686.27, 4388.35, 6000, 6845.79; 0.70, 0.80)(4033.98, 4740.38, 5547.62, 6000; 0.60, 0.50)
3	(4267.30, 4800.35, 6207.44, 6948.77; 0.70, 0.80)(4557.29, 5128.39, 5835.35, 6299.21; 0.60, 0.50)
4	(4557.59, 5188.88, 6791.74, 7467.19, 0.70, 0.80)(4887.08, 5654.70, 6409.50, 6787.73; 0.60, 0.50)
5	(5230.08, 5776.26, 7296.75, 7895.75; 0.70, 0.80)(5510.26, 6224.93, 6997.62, 7335.64; 0.60, 0.50)
6	(5903.97, 6525.15, 8024.19, 8552.33; 0.70, 0.80)(6153.92, 7012.16, 7775.57, 8076.76; 0.60, 0.50)

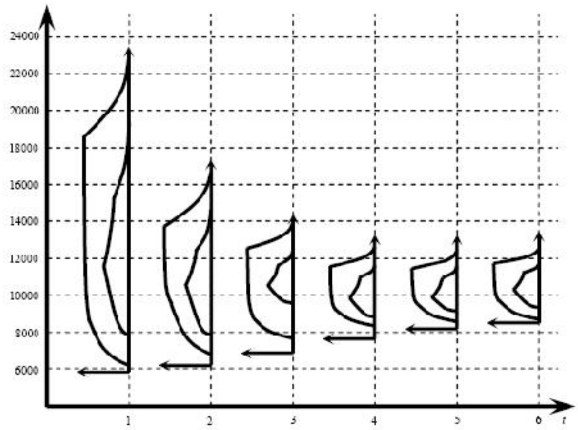

Fig 7.Trapezoidal Internal Type-2 Fuzzy Representation of Total Equivalent Uniform Annual Costs

Table 5 Trapezoidal Interval Type-2 Total Equivalent Uniform Annual Costs

t	Total Equivalent Uniform Annual Costs
1	(6240, 8540, 18520, 22860; 0.70, 0.90) (7980, 11540, 15200, 19420; 0.60, 0.50)
2	(6907.45, 9477.28, 13820.38, 16607.85; 0.70, 0.80)(7972.04, 10668.08, 12385.71, 14471.14; 0.60, 0.50)
3	(7710.61, 8975.56, 12516.33, 13964.38; 0.70, 0.80)(9637.78, 10578.05, 11695.59, 12397.80; 0.60, 0.50)
4	(8296.70; 9242.30, 11631.80, 12653.66; 0.70, 0.80)(8876.79, 9885.21, 11009.19, 11572.23; 0.55, 0.50)
5	(8599.48, 9337.20, 11457.90, 12265; 0.70, 0.80)(9071.21, 9981.77, 10954.58, 11394.19; 0.60, 0.50)
6	(8765.40, 9575.59, 11672.57, 12409.69; 0.70, 0.80)(9204.36, 10256.89, 11219.68, 11622.41; 0.60, 0.50)

To initiate the defuzzification process, we first degrade the results calculated in Table 5 to ordinary type-1 fuzzy sets as follows, using equations (11), (12), (13) and (14);

$$T(EUAC)_1^{op} = 14546.43, \quad T(EUAC)_2^{op} = 11864.20,$$

$$T(EUAC)_3^{op} = 10895.53, \quad T(EUAC)_4^{op} = 10528.50,$$

$$T(EUAC)_5^{op} = 10475.80, T(EUAC)_6^{op} = 10666.49$$

$$T(EUAC)_1^{pe} = 13275.22, \quad T(EUAC)_2^{pe} = 11231.16,$$

$$T(EUAC)_3^{pe} = 11015.77, \quad T(EUAC)_4^{pe} = 10304.63,$$

$$T(EUAC)_5^{pe} = 10300.28, T(EUAC)_6^{pe} = 10525.17$$

$$T(EUAC)_1^{re} = 13468.64, \quad T(EUAC)_2^{re} = 11356.26,$$

$$T(EUAC)_3^{re} = 10832.55, \quad T(EUAC)_4^{re} = 10354.21,$$

$$T(EUAC)_5^{re} = 10315.43, T(EUAC)_6^{re} = 10523.41$$

$$T(EUAC)_1^{wa} = 13565.52, T(EUAC)_2^{wa} = 11419.69,$$

$$T(EUAC)_3^{wa} = 10741.16, T(EUAC)_4^{wa} = 10379.21,$$

$$T(EUAC)_5^{wa} = 10323.52, T(EUAC)_6^{wa} = 10523.30$$

The corresponding modes are calculated for each case by using the equations (15) thru (26). As shown in Table 6, the economic lifetime value is year 5 for each 4 cases. Before discusing the findingsand examining the results in more detail, we have conducted a sensitivity analysis over the discount rate and the initial investment in order to figure out how EUAC figures react on their changes.

Table 6 Minimum Total EUAC's

Minimum Total EUAC	Value	t^*
$min\, T(EUAC)_t^{op} = T(EUAC)_5^{op}$	10475.80	5
$min\, T(EUAC)_t^{pe} = T(EUAC)_5^{pe}$	10300.28	5
$min\, T(EUAC)_t^{re} = T(EUAC)_5^{re}$	10315.43	5
$min\, T(EUAC)_t^{wa} = T(EUAC)_5^{wa}$	10323.52	5

VII. SENSITIVITY ANALYSIS

When we augment the discount rate per 25% the economic lifetime values intend to diminish. But we can conlude that the sensitivity of economic lifetime values are higher when the discount rate is decreasing versus the increase.The initial investment figure was a crisp value, as expected economic lifetime presented a positive correlation over the initial investment changes.As it is observed in Fig.5, the support set of type-2 fuzzy capital recovery is larger in early periods than the late periods, there exist two reasons for this difference; first, the effect of fuzziness of the early salvage values, and second, the higher fuziness of compound interest factor based on type-2 fuzzy discount rate. The sensitivity analysis of the type-2 fuzzy discount rate also revealed that eventhough change of type-2 fuzzy discount rate is positively correlated with type-2 fuzzy capital recovery figures, the difference between the supports of early and late figures are always protected. This would be the same case for initial investment changes over type-2 fuzzy capital recovery figures. On the other hand, type-2 fuzzy discount rate has a negative correlation over the supports of equivalent uniform annual operational costs. If the discount rate increases the support set of early periods are decreasing, while support sets of later periods are increasing, it is true that the fuzziness over costs would increase with higher fuzzy discount rate. Table 7 and 8 present economic lifetime values based on sensitivity analysis of the discount rate and initial investment, respectively.

Table 7 Economic Lifetime Values based on Sensivity Analysis of the Discount Rate

Value of $\tilde{\tilde{R}}$	Op	Pe	Re	Wa
$\tilde{\tilde{R}} \times 0.50$	4	4	4	4
$\tilde{\tilde{R}} \times 0.75$	5	4	5	5
$\tilde{\tilde{R}} \times 1.00$	5	5	5	5
$\tilde{\tilde{R}} \times 3.00$	6	5	5	5
$\tilde{\tilde{R}} \times 3.25$	6	6	6	6

Table 8 Economic Lifetime Values based on Sensitivity Analysis of Initial Investment

Value of $\tilde{\tilde{I}}$	Op	Pe	Re	Wa
$\tilde{I} \times 0.70$	3	1	1	1
$\tilde{I} \times 0.75$	4	2	4	4
$\tilde{I} \times 0.85$	4	4	4	4
$\tilde{\tilde{I}} \times 1.00$	5	5	5	5
$\tilde{I} \times 1.30$	5	5	5	5
$\tilde{\tilde{I}} \times 1.40$	6	5	5	6
$\tilde{\tilde{I}} \times 1.50$	6	6	6	6

CONCLUSION

As we have stated earlier, as the technology is progressing fast, the replacement decisions would become more important. Since the pattern and the replacement motivation behind of the decision makers have changed, replacement decisions should be taken into consideration in depth.In this paper, we have developed interval type-2 fuzzy framework for replacement analysis. Decision makers would benefit the advantages of type-2 fuzzy sets instead of using ordinary membership functions. Beside the development of this framework, we claim that the study would also visualize the behavior of cash flows by presenting the generalized replacement analysis diagrams. The graphical representation of interval type-2 fuzzy sets would be a leading visualized tool for decision makers.

The numerical example applied dealed with not only with the calculation of type-2 fuzzy sets and the defuzzification of them but also it has been followed by a sensitivity analysis. For further studies, new methods and frameworks should be developed for unlike replacement analysis where challengers and defenders are based on different technology generations.

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