TRAFFIC CONGESTION PROBLEM OF ROAD NETWORKS IN KOTA KINABALU VIA NETWORK GRAPH

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Abstract - KotaKinabalu is one of the major cities in Sabah which has traffic congestion due to abundance of private vehicle and slow improvement of traffic facilities. Traffic congestion is an urban mobility problem that worsens traffic flow and cause economic hindrance. Traffic volume and capacity are closely related to traffic congestion because once thevolumes exceed the capacity, the traffic congestion will occur. The scope of the study is a network from Indah Permai (IP) to Kota Kinabalu International Airport (KKIA).The objectives of this study are to find maximum flow, bottleneck and shortest path. Hence, capacity and distance of the routes within the scope were obtained from Dewan Bandaraya Kota Kinabalu (DBKK) and Google Map. Capacitated and weighted network graph were formed with all the obtained data. Next, Ford-Fulkerson algorithm, Max flow-Min cut theorem and Dijkstra's algorithm were applied to solve the network graph. The maximum flow and the shortest path problem were formulated intoa linear programming(LP) model, and solved by using the excel solver in Microsoft Excel. From theresults, the traffic congestion problem was minimized and traffic flow became smooth.

Keywords - Traffic Congestion, Maximum flow, Bottleneck, Shortest Distance, Linear ProgrammingModel

I. INTRODUCTION

Traffic congestion is a common traffic problem for the entire world, occurring from year to year. There are several reasons that cause traffic congestions like bottlenecks, accidents, road conditions, road facilities, driver's driving behaviors, unrestricted owning of vehicles and so forth. Traffic jam causes the traffic speeds slower, long trips time and long queues on the traffic. These phenomena are affecting the economic productivity and wasting of fuels and time. Apart from economic issues, traffic congestionsalso affected the quality of life and polluted the environment. Traffic congestion can be categorized into two types.

The first type is recurring congestion which used to happen in the area of Central Business District (CBD)during peak hours of weekdays. Non-recurring congestion is the second type of traffic congestion. It used to happen at an unexpected conditionsuch as accidents, sudden road closures, and maintenance that slow the traffic flow. Moreover, non-recurring congestion is an unpredictable and troublesome traffic problem because it reduces the roadway capacity [1]. In this study, recurring congestion are considered and discussed.

Traffic congestion problem has causing a major economic hindrance which state by World Bank in Borneo Post [2].The factors that contributing the traffic congestion were the abundance of private vehicles on the road and inefficient public transportation in Kota Kinabaluthat stated inBernama[3].Capacity is closely related to the maximum flow. If there is decrement of road capacity, it will cause the maximum flow decrease. If intersections experience decrease of maximum flow, then a long queue and the dramatic drop of vehicle's speed in traffic.

Hence, the objectivesof this study are to find the maximum flow of the desired routesas well as their bottlenecks, and determine the shortest path to reach the destination within the selected scope site.Ford-Fulkerson Algorithmcomputethe maximum flow, while theMaximum Flow and Minimum Cut Theorem identify thebottlenecks, and Dijkstra's Algorithm is used to identify the shortest path.

II. RELATED WORK

Implementation of Genetic Algorithm withinan urban traffic light intersection to optimizetraffic flow in Kota Kinabalu, Sabah was studied [4]. The increase of on-road vehiclesworsened congestion problems in Kota Kinabalu. Traffic light systems were built to control and ensure the smoothness of traffic flows at the intersections. However, traffic light systemcannot afford the increases of traffic flow and caused the long queue and congestion at intersection. The suggested solution was rebuilt of new traffic infrastructure like new roads and lanes, but it became more difficult due to the limited land available. Hence, the better solution to optimize the traffic flow was to examine and create a traffic light controller.

The data like queue length, green time, cycle time and amber time were observed and studied through simulations. Genetic algorithm was selected to find the optimized solution of traffic flow. Throughout the simulation results, it showed that the Genetic algorithm gives fast and good response to the change of queue length at the intersection.

The maximum flow problem and solution algorithm,Ford-Fulkerson algorithm in Ethiopian Airlines was investigated [5]. The maximum flow problem was solved by Ford-Fulkerson algorithm, the obtained maximum flow value was the same, but the number of augmenting paths, and flow of augmenting path might be different. It meant that the maximum flow value of the maximum problem was unique but it could have different augmenting path and different number of augmenting path.

The maximum flow with speed dependent capacities was applied in Bangkok traffic road networks [6].A traffic maximum flow problem had arcs represented as capacity of road (maximum vehicles that pass through per hour) that were functions of the traffic speed (kilometer per hour) and traffic density (vehicles per kilometer). In estimating road capacities, an empirical data on safe vehicle separations with a given speed were used. In this paper, the maximum flow problem with multiple source and sink node andspeed-dependent capacitieswas solved by amodified version of the Ford-Fulkerson algorithm. In overall, it found that30km/hour of speed is the maximum speed on traffic in order to have a safe traffic flow.

A method of path selection in the graph was presented [7]. In this paper,Dijkstra's algorithm was applied in maritime sector network graph to find additional paths among nodes. Since it involved a single criterion, therefore the shortest path was not always the best alternative path. Hence, other parameters such as average time travel, number of indirect vertices and other safety precautions aspects were calculated. Multi-criteria decision makingwas used in this study for selecting one desirable path from several paths. Dempster-Shafer theory was a method that could be applied tocombine data and evidences.

Ford-Fulkerson Maximum Flow procedure might unable to terminate the simplest and smallest network [8]. Ford-Fulkerson is a labeling method that can always terminate networks graph that have rational capacity of the edges. However, it might fail to terminate in the sense that it has an infinite sequence of flow augmentations.The results suggested that network with real-valued capacities contain the subgraph homeomorphic and irrational capacities. Therefore, Ford-Fulkerson algorithm might fail to terminate it.

Highway capacity was computed by the maximum flow algorithm and fundamental theory of highway traffic was studied [9]. In the traffic route map analysis, road capacity refers to the maximum number of vehicles at particular paths or edges. Multi point and multi destination traffic capacity network was converted into a single source node and sink node network problem. Network simplification process was performed to obtain the maximum flow of the network. It concluded that the results were the same as the results of the labeling method. Contribution of paper [9] was to transform a highway network capacity into precisemathematical model and solved by maximum flow algorithm such as Ford-Fulkerson Algorithm.

A research on method of identification bottleneck of traffic network via Max-flow Min-cut Theorem was performed [10]. It allowed the weak section of the road to be identified and provided a solution for the traffic problem. Before proceed to bottleneck identification, a traffic network with a map of graph must be formed.Then, Max-flow Min-cut Theorem was implemented to find out the bottleneck of the whole traffic network. The minimum cut of this theorem is the maximumflow of the whole road network. The identified weak parts of the road allow traffic planar to know that which parts of the road needed to be widened. The results showed that bottleneck depended on Max-flow Min-cut Theorem can be identified effectively.

An applied minimum-cut maximum-flow using cut set of a weighted graph on the traffic flow network [11]. A capacitated graph with a real number of capacity served as a structural model in transportation. The traffic control strategy of minimal cut and maximum flow was to minimize number of edges in network and maximum capacity of vehicles which can be moved through these edges. The technique of minimal cut in traffic network allowedshortening of the waiting time of traffic participants, and providing a smooth and uncongested traffic flow.

III. METHODOLOGY

3.1 Research Design

Figure 2 and 3 below show the research design to solve maximum flow problem and shortest path problem. Traffic congestion in Kota Kinabalu is discussed. In traffic congestion, there are two different types of it which are namely recurring and non-recurring congestion. Recurring congestion is focused in this study. Directed network graph with a single source and sink is used. In Figure 2, the algorithm that is used to solve maximum flow problem is Ford- Fulkerson algorithm followed by Max Flow & Min Cut Theorem. The results of the maximum flow problem are maximum flow and bottleneck of the network. According to Figure 3,Dijkstra's algorithm is utilized in the network in Kota Kinabalu to identify shortest path.

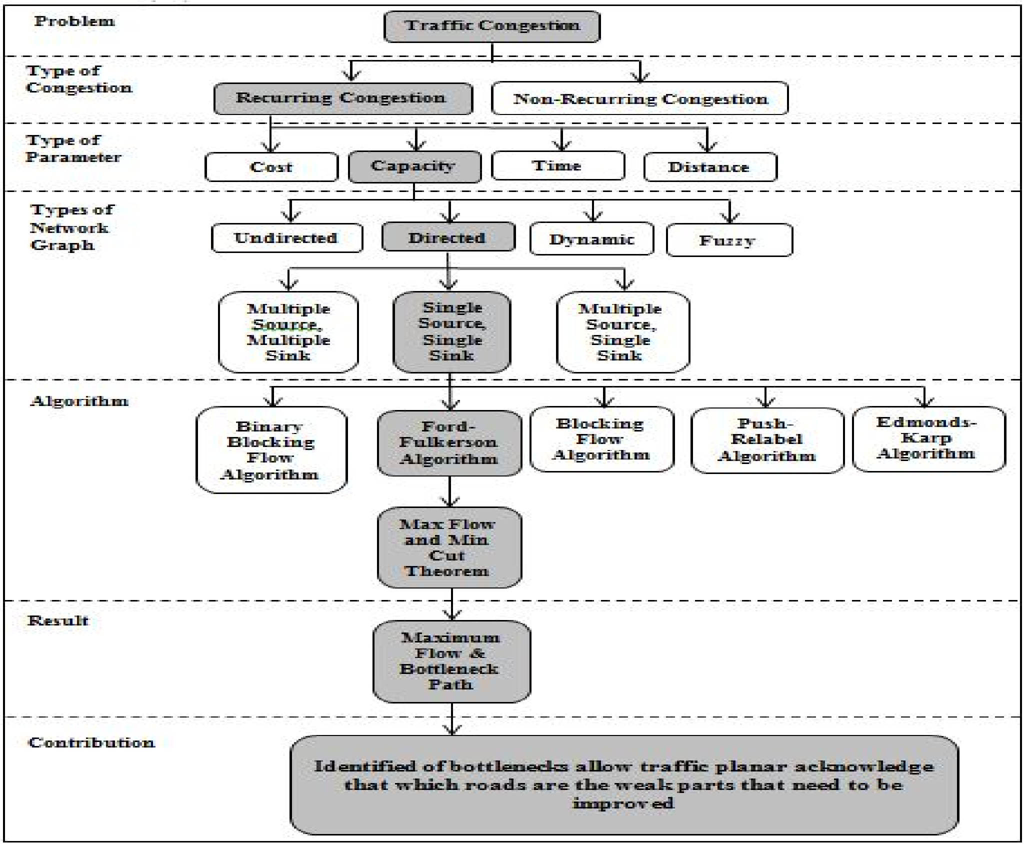

Figure 2: Research Design for Maximum Flow Problem

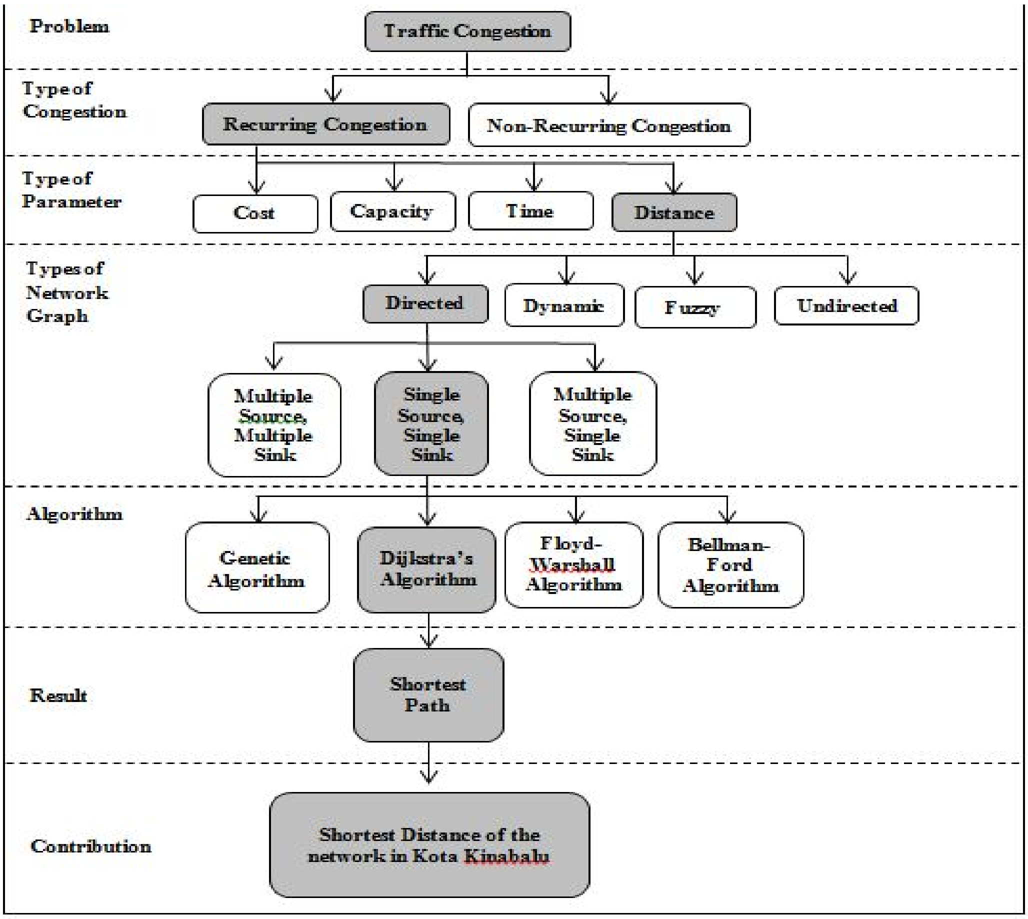

Figure 3: Research Design for Shortest Path Problem

3.2 Network Graph

Network is formed with edges that are connected with nodes. Capacitated network graph and weighted network graph are needed in this study to get the shortest path and maximal flow. First, a capacitated network graph formulates with all the edges. Each of the edges has a non-negative capacity, $c(u,v) \geq 0$ and flows $f(u,v)$ that cannot be more than capacity of the edge. The source node, s and sink node, t of a network are starting point, and ending point respectively. A capacitated network must fulfill the conditions below:

1. Capacity constraint, $\forall(u,v) \in E\ f(u,v) \leq c(u,v)$ which flow of the edges must not exceed its own capacity.
2. Skew symmetry, $\forall u,v \in V, f(u,v) = -f(v,u)$ which net flow from u to v and from v to u must be opposite to each other.
3. Flow conservation constraints, $\forall u \in V: u \neq s\ and\ u \neq t => \sum_{(s,u)\in E} f(s,u) = \sum_{(v,t)\in E} f(v,t)$ whichthe net flow to a node is zero except source node and sink node and the flow from the source node must be equal to the flow at the sink node.

Weighted network graph is a network graph that formulates by the edges with the non-negative distances. It is almost the same as the capacitated network graph and the only thing different is the parameter of edges [12].

3.3 Ford-Fulkerson Algorithm

Maximal flow in a capacitated flow network is the total flow from a source node to a sink node. First, find an augmenting path from the source node to the sink node. After the formation of augmentation path, compute the bottleneck capacity. Lastly, augment each edge and the total flow until the capacity of sink node reaches maximum [13].

3.4 Dijkstra's Algorithm

First, assign to every node a conditional distance value. Then, label the distance of the source node as zero and assign infiniteto all other nodes.Minimize the cost for each node in every following step. Label starting node as permanentand set it as current node. Then, temporarilylabel the new distanceof the unvisited adjacent nodesthat can be reaching from the current node with the least value.Once the adjacent nodes of the current node are considered, and then mark it as permanent. Then select a node from the temporary label adjacent nodes that has the smallest distance and repeat the previous step. End the algorithm once there are no possibilities to improve it further [14].

3.5 Maximum Flow and Minimum Cut Theorem

The minimum capacity of an (s,t)-cut is equal to the maximum value of a flow.

$$\max\{val(f) \mid f \text{ is a flow}\} = \min\{cap(S,T) \mid (S,T) \text{is an } \{s,t\} - \text{cut}\}$$

The bottleneck paths is the total maximum flow of the whole network graph.

3.5 Linear Programming Formulation for Maximum Flow Problem

$$\text{Maximize } x_{ts},$$

Subject to:

$$\sum_j x_{ij} - \sum_k x_{ki} = 0 \ \ (i = 1,2,\dots,n)$$

$$0 \leq x_{ij} \leq u_{ij} (i = 1,2,\dots,n; j = 1,2,\dots,n)$$

x_{ij}stand for the flow from node i to node j

x_{ts}stand for the amount of material send from node s to node t

u_{ij}stand for maximum flow of the given data

3.6 Linear Programming Formulation for Shortest Path Problem

$$\text{Minimize } z = \sum_i \sum_j c_{ij} x_{ij}$$

Subject to:

$$\sum_j x_{ij} - \sum_k x_{ki} = \begin{cases} 1 & \text{if } i = s \text{ (starting node)} \\ 0 & \text{otherwise} \\ -1 & \text{if } i = t \text{(end node)} \end{cases}$$

$$x_{ij} \geq 0 \text{ for all arcs } i - j \text{ in the network}$$

x_{ij}stand for the flow from node i to node j

IV. SCOPE SITE

The scope of this study is the network from Indah Permai-IP (source node) to Kota Kinabalu International Airport-KKIA (sink node) where all the routes between source and sink node are established. In Figure 1, the red nodes are the selected major intersections which are assigned as the nodes of the network graph. The yellow lines that connected those red nodes are path that connected the intersections. This scope area in is selected because this area is part of a central business district for Kota Kinabalu where the demand of traffic is higher than the other locations.

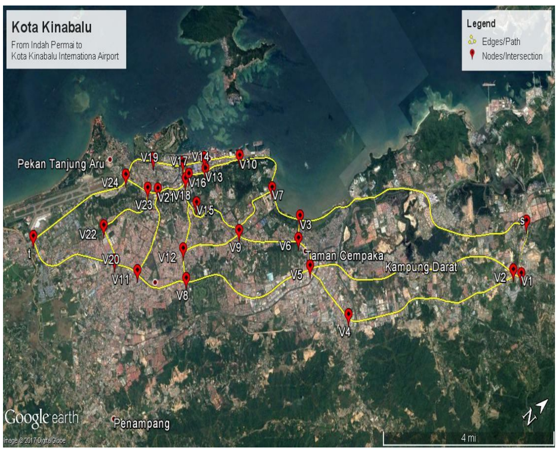

Figure 1: Scope of study (Within Kota Kinabalu Area)

V. DATA COLLECTION

Maximum flow, bottleneck and shortest path were the focus of this study. Hence, data such as distance, capacity and road direction of the routes within selected scopes were needed. These data were collected from Dewan Bandaraya Kota Kinabalu (DBKK), a city council in Kota Kinabalu and Google Maps. Direct empirical method was used in paper [15] for capacity estimation, but in this paper, traffic signal timing manual was used to get the capacity of the road [16].

To form a network graph, nodes and edges were needed. Therefore, the intersections were appointed as the nodes of the network graph and the paths that connected between the intersections were edges. Then, distance and capacity were assigned to all the edges to form a directed network graph. Lastly, Network algorithms, maximum flow algorithm, shortest path algorithmand Maximum Flow and Minimum Cut Theorem, were applied into the directed network graphto get outputs of this study.

No	Location Name	From	To
1	JalanSepanggar	S	V1
2	Jalan UMS	S	V3
3	JalanTuaran 1	V1	V2
4	JalanTuaran 2	V2	V4
5	JalanTuaran Bypass (North)	V2	V5
6	JalanTuaran Bypass (South)	V5	V2
7	JalanTuaran 3	V4	V5
8	Jalan Lintas 1	V5	V8
9	JalanTuaran 4(South)	V5	V6
10	JalanTuaran 4 (North)	V6	V5
11	JalanPasir (North)	V3	V6
12	JalanPasir (South)	V6	V3
13	JalanTunFuad Stephen 1	V3	V7
14	JalanTunFuad Stephen 2	V7	V10
15	JalanIstiadat& JalanKompleksSukan (North)	V7	V9
16	JalanIstiadat& JalanKompleksSukan (South)	V9	V7
17	JalanTuaran 5 (South)	V9	V6
18	JalanTuaran 5 (North)	V6	V9
19	JalanDamai (North)	V9	V12
20	JalanDamai (South)	V12	V9
21	JalanTuaran 6 (North)	V9	V15
22	JalanTuaran 6 (South)	V15	V9
23	Jalan KK Bypass 2 (North)	V10	V13
24	Jalan KK Bypass 2 (South)	V13	V10
25	JalanTunku Abdul Rahman 1 (North)	V13	V16
26	JalanTunku Abdul Rahman 1 (South)	V16	V13
27	JalanLaimanDiki	V13	V14
28	JalanTunRazak	V10	V14
29	Jalan Coastal 1	V14	V17
30	JalanKolam 1 (North)	V12	V15
31	JalanKolam 1 (South)	V15	V12
32	JalanKolam 2 (North)	V8	V12
33	JalanKolam 2 (South)	V12	V8
34	Jalan Lintas 2	V8	V11
35	Jalan Lintas 3	V11	V20
36	JalanPenampang (North)	V11	V21
37	JalanPenampang (South)	V21	V11
38	JalanTuaran 7 (North)	V15	V18
39	JalanTuaran 7 (South)	V18	V15
40	JalanTunku Abdul Rahman 2 (North)	V16	V18
41	JalanTunku Abdul Rahman 2 (South)	V18	V16
42	JalanNenas (North)	V15	V16
43	JalanNenas (South)	V16	V15
44	JalanKemajuan	V18	V17
45	JalanTunku Abdul Rahman 3 (North)	V18	V21
46	JalanTunku Abdul Rahman 3 (South)	V21	V18
47	JalanPintas 1 (North)	V20	V22
48	JalanPintas 1 (South)	V22	V20
49	JalanPintas 2 (North)	V22	V23
50	JalanPintas 2 (South)	V23	V22
51	JalanTunku Abdul Rahman 4 (North)	V21	V23
52	JalanTunku Abdul Rahman 4 (South)	V23	V21
53	Jalan Coastal 2	V17	V19
54	JalanSembulan (North)	V19	V23
55	JalanSembulan (South)	V23	V19
56	Jalan Coastal 3	V19	V24
57	Jalan Mat Salleh (North)	V23	V24
58	Jalan Mat Salleh (South)	V24	V23
59	JalanKepayan	V24	T
60	Jalan Lintas 4	V20	T

Table 1: The selected routes from IP to KKIA

No	From	To	Capacity	Distance (Meter)
1	S	V1	914	1600
2	S	V3	3103	11500
3	V1	V2	1394	350
4	V2	V4	1905	6700
5	V2	V5	1151	8200
6	V5	V2	1151	8200
7	V4	V5	2161	2000
8	V5	V8	2877	5100
9	V5	V6	2559	1000

10	V6	V5	2832	1000
11	V3	V6	1533	600
12	V6	V3	2500	600
13	V3	V7	2000	1500
14	V7	V10	1948	2400
15	V7	V9	846	2100
16	V9	V7	2120	2100
17	V9	V6	1902	2300
18	V6	V9	1648	2300
19	V9	V12	1687	2200
20	V12	V9	1289	2200
21	V9	V15	1807	2200
22	V15	V9	813	2200
23	V10	V13	1897	1400
24	V13	V10	1394	1400
25	V13	V16	1482	600
26	V16	V13	2575	600
27	V13	V14	809	290
28	V10	V14	1995	1600
29	V14	V17	2223	800
30	V12	V15	2161	1400
31	V15	V12	2161	1400
32	V8	V12	1902	850
33	V12	V8	2877	850
34	V8	V11	2223	2000
35	V11	V20	1561	900
36	V11	V21	1388	2600
37	V21	V11	1514	2600
38	V15	V18	3600	800
39	V18	V15	2726	800
40	V16	V18	3757	150
41	V18	V16	1482	150
42	V15	V16	1482	1100
43	V16	V15	1877	1100
44	V18	V17	2262	450
45	V18	V21	1078	1100
46	V21	V18	3757	1100
47	V20	V22	1251	1100
48	V22	V20	2483	1100
49	V22	V23	2200	2300
50	V23	V22	1251	2300
51	V21	V23	2200	350
52	V23	V21	1948	350
53	V17	V19	2559	1100
54	V19	V23	3300	950
55	V23	V19	1419	950
56	V19	V24	4428	1400
57	V23	V24	895	900
58	V24	V23	2200	900
59	V24	T	2426	4000
60	V20	T	1591	3600

Table 2: Capacity and Distance of the selected routes from IP to KKIA

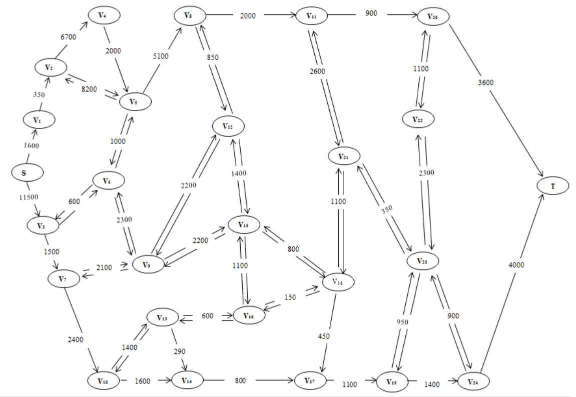

Figure 4: Weighted Directed Network Graph from IP to KKIA

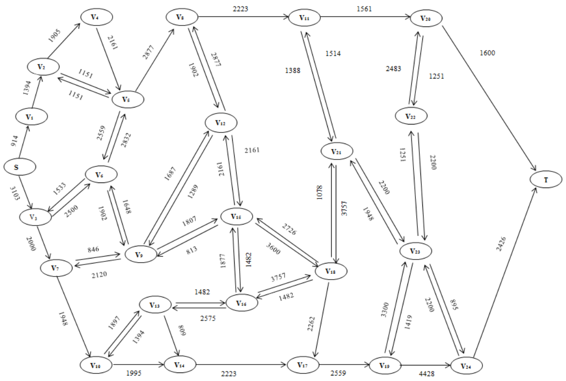

Figure 5: Capacitated Directed Network Graph from IP to KKIA

Weighted directed network graph in figure 4 and capacitated directed network graph in figure 5 were formed by the data in Table 2.

5.1 Define Decision Variable for formulation of LP Model

A series of consecutive integers is used to assign number to the node of the network. The node numbers allow the identification of the decision variable much more convenient. The decision variable needed to formulate the Linear Programming (LP) Model for the Shortest Path Problem and Maximum Flow Problem.

For each arc in a network flow model, decision variable must define as :

$$X_{ij} = \text{the number of capacity and distance from node } i \text{ to node } j$$

In order to formulate LP model, the following 60 decision variables is needed which presented below.

X_{s1} = *capacity and distance value of node s to node* 1

X_{s3} = *capacity and distance value of node s to node* 3

X_{12} = *capacity and distance value of node* 1 *to node* 2

X_{24} = *capacity and distance value of node* 2 *to node* 4

X_{45} = *capacity and distance value of node* 4 *to node* 5

X_{58} = *capacity and distance value of node* 5 *to node* 8

X_{811} = *capacity and distance value of node* 8 *to node* 11

X_{20t} = *capacity and distance value of node* 20 *to node t*

X_{37} = *capacity and distance value of node* 3 *to node* 7

X_{710} = *capacity and distance value of node* 7 *to node* 10

X_{24t} = *tcapacity and distance value of node* 24 *to node t*

X_{25} = *capacity and distance value of node* 2 *to node* 5

X_{52} = *capacity and distance value of node* 5 *to node* 2

X_{56} = *capacity and distance value of node* 5 *to node* 6

X_{65}
$= capacity\ and\ distance\ value\ of\ node\ 6\ to\ node\ 5$
X_{63}
$= capacity\ and\ distance\ value\ of\ node\ 6\ to\ node\ 3$
X_{36}
$= capacity\ and\ distance\ value\ of\ node\ 3\ to\ node\ 6$
X_{69}
$= capacity\ and\ distance\ value\ of\ node\ 6\ to\ node\ 9$
X_{96}
$= capacity\ and\ distance\ value\ of\ node\ 9\ to\ node\ 6$
X_{79}
$= capacity\ and\ distance\ value\ of\ node\ 7\ to\ node\ 9$
X_{97}
$= capacity\ and\ distance\ value\ of\ node\ 9\ to\ node\ 7$
X_{915}
$= capacity\ and\ distance\ value\ of\ node\ 9\ to\ node\ 15$
X_{159}
$= capacity\ and\ distance\ value\ of\ node\ 15\ to\ node\ 9$
X_{129}
$= capacity\ and\ distance\ value\ of\ node\ 12\ to\ node\ 9$
X_{912}
$= capacity\ and\ distance\ value\ of\ node\ 9\ to\ node\ 12$
X_{812}
$= capacity\ and\ distance\ value\ of\ node\ 8\ to\ node\ 12$
X_{128}
$= capacity\ and\ distance\ value\ of\ node\ 12\ to\ node\ 8$
X_{1516}
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X_{1816}
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X_{1618}
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X_{2118}
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X_{1821}
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X_{1121}
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X_{2111}
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X_{2123}
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X_{2322}
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X_{1923}
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X_{2319}
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X_{2324}
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X_{2423}
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X_{2022}
$= capacity\ and\ distance\ value\ of\ node\ 20\ to\ node\ 22$
X_{2220}
$= capacity\ and\ distance\ value\ of\ node\ 22\ to\ node\ 20$
X_{1120}
$= capacity\ and\ distance\ value\ of\ node\ 11\ to\ node\ 20$
X_{1014}
$= capacity\ and\ distance\ value\ of\ node\ 10\ to\ node\ 14$
X_{1817}
$= capacity\ and\ distance\ value\ of\ node\ 18\ to\ node\ 17$
X_{1719}
$= capacity\ and\ distance\ value\ of\ node\ 17\ to\ node\ 19$
X_{1924}
$= capacity\ and\ distance\ value\ of\ node\ 19\ to\ node\ 24$
X_{1417}
$= capacity\ and\ distance\ value\ of\ node\ 14\ to\ node\ 17$
X_{1013}
$= capacity\ and\ distance\ value\ of\ node\ 10\ to\ node\ 13$
X_{1310}
$= capacity\ and\ distance\ value\ of\ node\ 13\ to\ node\ 10$
X_{1215}
$= capacity\ and\ distance\ value\ of\ node\ 12\ to\ node\ 15$
X_{1512}
$= capacity\ and\ distance\ value\ of\ node\ 15\ to\ node\ 12$

5.2A Linear Programming Model for Maximum Flow Problem

The maximum flow can be obtained by solving a Linear Programming Model from node s to node t, given appropriate upper bounds on each arc and usual balance-of-flow constraints.

$Max : X_{ts}$
Subject to:
$Node\ s: X_{ts} - X_{s1} - X_{s3} = 0$
$Node\ 1: X_{S1} - X_{12} = 0$

$Node\ 2: X_{12} + X_{52} - X_{24} - X_{25} = 0$
$Node\ 3: X_{s3} + X_{63} - X_{37} - X_{36} = 0$
$Node\ 4: X_{24} - X_{45} = 0$
$Node\ 5: X_{45} + X_{25} + X_{65} - X_{52} - X_{56} - X_{58} = 0$
$Node\ 6: X_{56} + X_{36} + X_{96} - X_{65} - X_{63} - X_{69} = 0$
$Node\ 7: X_{37} + X_{97} - X_{79} - X_{710} = 0$
$Node\ 8: X_{58} + X_{128} - X_{812} - X_{811} = 0$
$Node\ 9: X_{69} + X_{79} + X_{129} + X_{159} - X_{915} - X_{912} - X_{96} - X_{97} = 0$
$Node\ 10: X_{710} + X_{1310} - X_{1014} - X_{1013} = 0$
$Node\ 11: X_{811} + X_{2111} - X_{1121} - X_{1120} = 0$
$Node\ 12: X_{812} + X_{1512} + X_{912} - X_{129} - X_{1215} - X_{128} = 0$
$Node\ 13: X_{1013} + X_{1316} - X_{1314} - X_{1310} - X_{1316} = 0$
$Node\ 14: X_{1014} + X_{1314} - X_{1417} = 0$
$Node\ 15: X_{1215} + X_{915} + X_{1615} + X_{1815} - X_{1518} - X_{1516} - X_{159} - X_{1512} = 0$
$Node\ 16: X_{1516} + X_{1316} + X_{1816} - X_{1618} - X_{1615} - X_{1613} = 0$
$Node\ 17: X_{1817} + X_{1417} - X_{1719} = 0$
$Node\ 18: X_{2118} + X_{1518} + X_{1618} - X_{1817} - X_{1816} - X_{1815} - X_{1821} = 0$

$Node\ 19: X_{1719} + X_{2319} - X_{1923} - X_{1924} = 0$
$Node\ 20: X_{1120} + X_{2220} - X_{20t} - X_{2022} = 0$
$Node\ 21: X_{1121} + X_{1821} + X_{2321} - X_{2123} - X_{2111} - X_{2118} = 0$
$Node\ 22: X_{2022} + X_{2322} - X_{2223} - X_{2220} = 0$
$Node\ 23: X_{2223} + X_{2123} + X_{1923} + X_{2423} - X_{2324} - X_{2319} - X_{2321} - X_{2322} = 0$
$Node\ 24: X_{2324} + X_{1924} - X_{2423} - X_{24t} = 0$
$Node\ t: X_{20t} + X_{24t} - X_{ts} = 0$

$0 \le X_{s1} \le 914, 0 \le X_{s3} \le 3103$
$, 0 \le X_{12} \le 1394, 0 \le X_{24} \le 1905$
$, 0 \le X_{45} \le 2161, 0 \le X_{25} \le 1151$
$, 0 \le X_{52} \le 1151, 0 \le X_{56} \le 2559$
$, 0 \le X_{65} \le 2832, 0 \le X_{58} \le 2877$
$, 0 \le X_{36} \le 2500, 0 \le X_{63} \le 1533$
$, 0 \le X_{37} \le 2000, 0 \le X_{710} \le 1948$
$, 0 \le X_{1014} \le 1995, 0 \le X_{79} \le 846$
$, 0 \le X_{97} \le 2120, 0 \le X_{69} \le 1902$
$, 0 \le X_{96} \le 1648, 0 \le X_{912} \le 1687$
$, 0 \le X_{129} \le 1289, 0 \le X_{812} \le 1902$
$, 0 \le X_{128} \le 2877, 0 \le X_{1215} \le 2161$
$, 0 \le X_{1512} \le 2161, 0 \le X_{915} \le 1807$
$, 0 \le X_{159} \le 813, 0 \le X_{1013} \le 1897$
$, 0 \le X_{1310} \le 1394, 0 \le X_{1314} \le 809$
$, 0 \le X_{1316} \le 1482, 0 \le X_{1613} \le 2575$
$, 0 \le X_{1516} \le 1482, 0 \le X_{1615} \le 1877$
$, 0 \le X_{1618} \le 3757, 0 \le X_{1816} \le 1482$
$, 0 \le X_{1518} \le 3600, 0 \le X_{1815} \le 2726$
$, 0 \le X_{1817} \le 2262, 0 \le X_{1417} \le 2223$
$, 0 \le X_{1719} \le 2559, 0 \le X_{1924} \le 4428$
$, 0 \le X_{2319} \le 1419, 0 \le X_{1923} \le 3300$
$, 0 \le X_{2324} \le 895, 0 \le X_{2423} \le 2200$
$, 0 \le X_{2118} \le 3757, 0 \le X_{1821} \le 1078$
$, 0 \le X_{2123} \le 2200, 0 \le X_{2321} \le 1948$
$, 0 \le X_{2111} \le 1514, 0 \le X_{1121} \le 1388$

$, 0 \le X_{811} \le 2223, 0 \le X_{1120} \le 1561$
$, 0 \le X_{2022} \le 1251, 0 \le X_{2220} \le 2483$
$, 0 \le X_{2223} \le 2200, 0 \le X_{2322} \le 1251$
$, 0 \le X_{20t} \le 1600, 0 \le X_{24t} \le 2426$

5.3 A Linear Programming Model for Shortest Path Problem

The Linear Programming Model to minimize the distance of travel by using balance-of-flow is represented as below.

$$
\begin{aligned}
MIN: 16000X_{s1} &+ 350X_{12} + 6700X_{24} + 2000X_{45} \\
&+ 8200X_{25} + 8200X_{52} \\
&+ 1000X_{56} + 1000X_{65} \\
&+ 600X_{63} + 600X_{36} \\
&+ 11500X_{s3} + 1500X_{37} \\
&+ 2400X_{710} + 2100X_{79} \\
&+ 2100X_{97} + 2300X_{69} \\
&+ 2300X_{96} + 5100X_{58} \\
&+ 850X_{812} + 850X_{128} \\
&+ 2000X_{811} + 2200X_{912} \\
&+ 2200X_{129} + 2200X_{915} \\
&+ 2200X_{159} + 1400X_{1215} \\
&+ 1400X_{1512} + 1100X_{1516} \\
&+ 1100X_{1615} + 800X_{1518} \\
&+ 800X_{1815} + 150X_{1618} \\
&+ 150X_{1816} + 600X_{1316} \\
&+ 600X_{1613} + 290X_{1314} \\
&+ 1400X_{1013} + 1400X_{1310} \\
&+ 1600X_{1014} + 800X_{1417} \\
&+ 450X_{1817} + 1100X_{1719} \\
&+ 1400X_{1924} + 950X_{2319} \\
&+ 950X_{1923} + 900X_{2324} \\
&+ 900X_{2423} + 1100X_{2118} \\
&+ 1100X_{1821} + 350X_{2123} \\
&+ 350X_{2321} + 2600X_{1121} \\
&+ 2600X_{2111} + 900X_{1120} \\
&+ 1100X_{2022} + 1100X_{2220} \\
&+ 2300X_{2223} + 2300X_{2322} \\
&+ 3600X_{20t} + 4000X_{24t}
\end{aligned}
$$

Subject to:

$Node\ S: -X_{s1} - X_{s3} = -1$
$Node\ 1: X_{s1} - X_{12} = 0$
$Node\ 2: X_{12} + X_{52} - X_{25} - X_{24} = 0$
$Node\ 3: X_{s3} + X_{63} - X_{36} - X_{37} = 0$
$Node\ 4: X_{24} - X_{45} = 0$
$Node\ 5: X_{45} + X_{25} + X_{65} - X_{52} - X_{56} - X_{58} = 0$
$Node\ 6: X_{56} + X_{36} + X_{96} - X_{65} - X_{63} - X_{69} = 0$
$Node\ 7: X_{37} + X_{97} - X_{79} - X_{710} = 0$
$Node\ 8: X_{58} + X_{128} - X_{812} - X_{811} = 0$
$Node\ 9: X_{69} + X_{79} + X_{129} + X_{159} - X_{96} - X_{97} - X_{912} - X_{915} = 0$
$Node\ 10: X_{710} + X_{1310} - X_{1013} - X_{1014} = 0$
$Node\ 11: X_{811} + X_{2111} - X_{1121} - X_{1120} = 0$
$Node\ 12: X_{812} + X_{912} + X_{1512} - X_{128} - X_{129} - X_{1215} = 0$
$Node\ 13: X_{1013} + X_{1613} - X_{1316} - X_{1310} - X_{1314} = 0$
$Node\ 14: X_{1014} + X_{1314} - X_{1417} = 0$
$Node\ 15: X_{1215} + X_{915} + X_{1615} + X_{1815} - X_{1512} - X_{1518} - X_{1516} - X_{159} = 0$
$Node\ 16: X_{1516} + X_{1316} + X_{1816} - X_{1618} - X_{1613} - X_{1615} = 0$
$Node\ 17: X_{1817} + X_{1417} - X_{1719} = 0$
$Node\ 18: X_{2118} + X_{1518} + X_{1618} - X_{1817} - X_{1821} - X_{1815} - X_{1816} = 0$
$Node\ 19: X_{1719} + X_{2319} - X_{1923} - X_{1924} = 0$
$Node\ 20: X_{1120} + X_{2220} - X_{2022} - X_{20t} = 0$
$Node\ 21: X_{1121} + X_{1821} + X_{2321} - X_{2111} - X_{2123} - X_{2118} = 0$
$Node\ 22: X_{2022} + X_{2322} - X_{2220} - X_{2223} = 0$
$Node\ 23: X_{2123} + X_{2223} + X_{1923} + X_{2423} - X_{2322} - X_{2321} - X_{2324} - X_{2319} = 0$
$Node\ 24: X_{1924} + X_{2324} - X_{2423} - X_{24t} = 0$
$Node\ t: X_{24t} + X_{20t} = 1$
$X_{ij} \ge 0\ for\ all\ i\ and\ j$

RESULTS AND DISCUSSIONS

6.1 Results of Maximum Flow Problem Using Algorithms

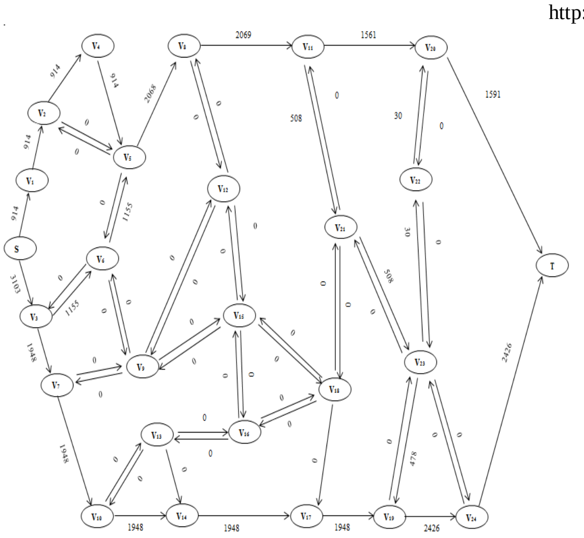

Figure 6: Optimal Solution of Maximum Flow Problem

In Figure 6,numbers of augmenting path were formed to find the total maximum flow. The first augmenting pathwas S → V1 → V2 → V4 → V5 → V8 → V11 → V20 → T (.Second augmenting path was S → V3 → V6 → V5 → V8 → V11 → V20 → T . Third augmenting path was S → V3 → V6 → V5 → V8 → V11 → V21 → V23 → V19 → V24 → T . Fourth augmenting path was S → V3 → V6 → V5 → V8 → V11 → V21 → V23 → V22 → V20 → T . Fifth augmenting path was S → V3 → V7 → V10 → V14 → V17 → V19 → V24 → T . The maximum flow of first, second, third, fourth and fifth augmenting paths were914, 647, 478, 30 and 1948 vehicles, respectively. Hence, 4017 of vehicles per hour was the total maximum flow of those augmenting paths.

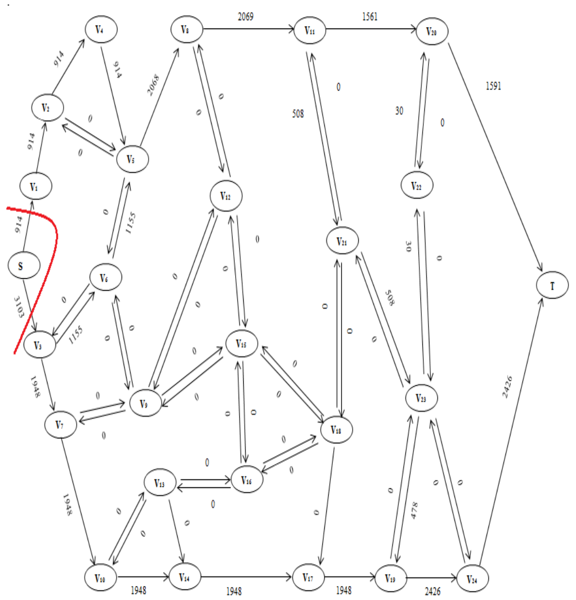

Figure 7: Output of Maximum Flow and Minimum Cut Theorem

The bottleneckpaths of the network have the same value as total maximum flow. Hence, S → V1 and S → V3are the bottleneck paths that showed in Figure 7. S → V1 wasJalanSepanggarand S → V3 was Jalan UMS.

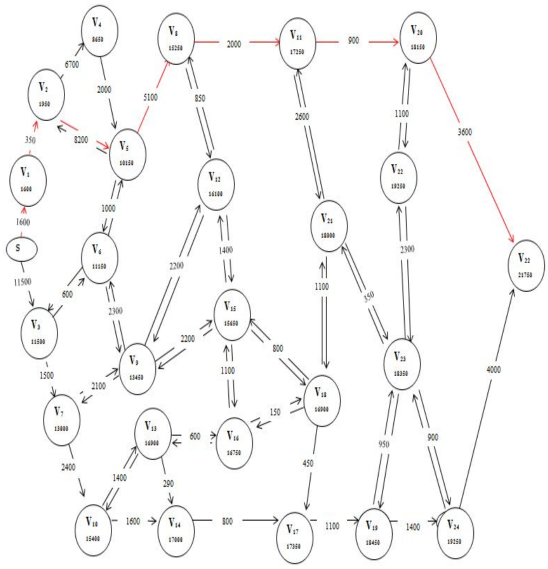

Figure 8: Output of Dijkstra's algorithm

From Figure 8, the shortest path in this weighted network graph was S → V1 → V2 → V5 → V8 → V11 → V20 → T. It was about 21.75kmfrom Indah Permai to Kota Kinabalu International Airport.

6.2 Excel Solver Output: Maximum Flow Problem

	From	To	Unit of Flow	≤	Capacity	Node	Net Flow	Supply/Demand
4	S	V1	914	≤	914	S	0	0
5	S	V3	3103	≤	3103	V1	0	0
6	V1	V2	914	≤	1394	V2	0	0
7	V2	V4	914	≤	1905	V3	0	0
8	V3	V6	1155	≤	1533	V4	0	0
9	V6	V3	0	≤	2500	V5	0	0
10	V2	V5	0	≤	1151	V6	0	0
11	V5	V2	0	≤	1151	V7	0	0
12	V3	V7	1948	≤	2000	V8	0	0
13	V7	V9	0	≤	846	V9	0	0
14	V9	V7	0	≤	2120	V10	0	0
15	V7	V10	1948	≤	1948	V11	0	0
16	V6	V9	0	≤	1902	V12	0	0
17	V9	V6	0	≤	1648	V13	0	0
18	V6	V5	1155	≤	2832	V14	0	0
19	V5	V6	0	≤	2559	V15	0	0
20	V4	V5	914	≤	2161	V16	0	0
21	V5	V8	2069	≤	2877	V17	0	0
22	V8	V12	0	≤	1902	V18	0	0
23	V12	V8	0	≤	2877	V19	0	0
24	V9	V12	0	≤	1687	V20	0	0
25	V12	V9	0	≤	1289	V21	0	0
26	V9	V15	0	≤	1807	V22	0	0
27	V15	V9	0	≤	813	V23	0	0
28	V15	V12	0	≤	2161	V24	0	0
29	V12	V15	0	≤	2161	T	0	0
30	V10	V13	0	≤	1897			
31	V13	V10	0	≤	1394			
32	V10	V14	1948	≤	1995			
33	V13	V14	0	≤	809			
34	V13	V16	0	≤	1482			
35	V16	V13	0	≤	2575			
36	V16	V18	0	≤	3757			
37	V18	V16	0	≤	1482			
38	V18	V21	0	≤	1078			
39	V21	V18	0	≤	3757			
40	V16	V15	0	≤	1877			
41	V15	V16	0	≤	1482			
42	V15	V18	0	≤	3600			
43	V18	V15	0	≤	2726			
44	V18	V17	0	≤	2262			
45	V11	V21	508	≤	1388			
46	V21	V11	0	≤	1514			
47	V20	V22	0	≤	1251			
48	V22	V20	30	≤	2483			
49	V8	V11	2069	≤	2223			
50	V11	V20	1561	≤	1561			
51	V21	V23	508	≤	2200			
52	V23	V21	0	≤	1948			
53	V23	V19	478	≤	1419			
54	V19	V23	0	≤	3300			

55	V14	V17	1948	≤	2223
56	V17	V19	1948	≤	2559
57	V19	V24	2426	≤	4428
58	V24	T	2426	≤	2426
59	V20	T	1591	≤	1600
60	V23	V24	0	≤	895
61	V24	V23	0	≤	2200
62	V23	V22	30	≤	1251
63	V22	V23	0	≤	2200
64	T	S	4017	≤	99999999
65					
66					
67	Maximum Flow		4017		

Figure 9: Maximum flow problem Excel Output

In Figure 9, column G represented the capacity for each edge. The objective function was the cell G67 which contained the formula of '=E64'. Cell E64 was the maximum flow from node "s" to node "t". The cell J4 to cell J29 represented the net flow which was the constraints cells. The units of flow, from cell E4 to cell E64, were the variable cells as shown in table 3. Those variable cells that equal to zero were the unutilized paths.

Key Cell Formulas		
Cell	Formula	Copied to
G67	=E64	-
J4	=SUMIF($C$4:$C$64,$I$4,$E$4:$E$64)-SUMIF($D$4:$D$64,$I$4,$E$4:$E$64)	J4:J29

Table 3: Formula of the cells in figure 9

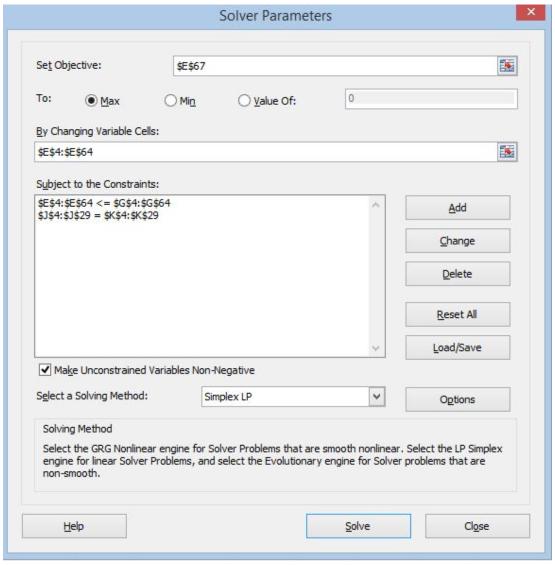

Figure 10: Solver parameters for maximal flow problem

For the solver parameters that shown in figure 10, the cell G67 was set as the Objective. The maximum button was chosen in order to maximize the maximum flow problem. The changing variable cells were the cells from E4 to E64. Next, constraints were the Unit of Flow (cell E4 to E64) less than or equal to Capacity (cells G4 to G64) and Net Flow (cells J4 to J29) must equal to the Supply/Demand (cells K4 to K29). Simplex Linear Programming was selected as the solving method. The augmenting path of the maximum flow path by excel solver might not be the same as the augmenting path of the Ford-Fulkerson algorithm. However, the maximum flow value for Ford-Fulkerson algorithm and excel outputs were expected to be the same [17].

6.2Excel Solver Output: Shortest Path Problem

	A	B	C	D	E	F	G	H	I	J
2										
3		From	To	On Route	=	Route Distance (Meter)		Node	Net Flow	Supply/Demand
4		S	V1	1	=	1600		S	1	1
5		S	V3	0	=	11500		V1	0	0
6		V1	V2	1	=	350		V2	0	0
7		V2	V4	0	=	6700		V3	0	0
8		V2	V5	1	=	8200		V4	0	0
9		V5	V2	0	=	8200		V5	0	0
10		V4	V5	0	=	2000		V6	0	0
11		V5	V8	1	=	5100		V7	0	0
12		V5	V6	0	=	1000		V8	0	0
13		V6	V5	0	=	1000		V9	0	0
14		V3	V6	0	=	600		V10	0	0
15		V6	V3	0	=	600		V11	0	0
16		V3	V7	0	=	1500		V12	0	0
17		V7	V10	0	=	2400		V13	0	0
18		V7	V9	0	=	2100		V14	0	0
19		V9	V7	0	=	2100		V15	0	0
20		V9	V6	0	=	2300		V16	0	0
21		V6	V9	0	=	2300		V17	0	0
22		V9	V12	0	=	2200		V18	0	0
23		V12	V9	0	=	2200		V19	0	0
24		V9	V15	0	=	2200		V20	0	0
25		V15	V9	0	=	2200		V21	0	0
26		V10	V13	0	=	1400		V22	0	0
27		V13	V10	0	=	1400		V23	0	0
28		V13	V16	0	=	600		V24	0	0
29		V16	V13	0	=	600		T	-1	-1
30		V13	V14	0	=	290				
31		V10	V14	0	=	1600				
32		V14	V17	0	=	800				
33		V12	V15	0	=	1400				
34		V15	V12	0	=	1400				
35		V8	V12	0	=	850				
36		V12	V8	0	=	850				
37		V8	V11	1	=	2000				
38		V11	V20	1	=	900				
39		V11	V21	0	=	2600				
40		V21	V11	0	=	2600				
41		V15	V18	0	=	800				
42		V18	V15	0	=	800				
43		V16	V18	0	=	150				
44		V18	V16	0	=	150				
45		V15	V16	0	=	1100				
46		V16	V15	0	=	1100				
47		V17	V18	0	=	450				
48		V18	V17	0	=	450				
49		V18	V21	0	=	1100				
50		V21	V18	0	=	1100				
51		V20	V22	0	=	1100				
52		V22	V20	0	=	1100				
53		V22	V23	0	=	2300				
54		V23	V22	0	=	2300				
55		V21	V23	0	=	350				
56		V23	V21	0	=	350				
57		V17	V19	0	=	1100				
58		V19	V23	0	=	950				
59		V23	V19	0	=	950				
60		V19	V24	0	=	1400				
61		V23	V24	0	=	900				
62		V24	V23	0	=	900				
63		V24	T	0	=	4000				
64		V20	T	1	=	3600				
65										
66		SHORTEST PATH:				21750				

Figure 11: Shortest path problem solved by using simplex linear programming in Microsoft excel

In Figure 11, column F represented the distance of the edge. The objective function was the cell F66 which contained the formula of '=SUMPRODUCT (D4:D64,F4:F64)'. The cell from I4 to I29 represented the net flow were the constraints cells which shown in Table 4. The 'On Route' from cell D4 to cell D64 were the variable cells. The number '1' showed in the column of 'on route' denoted for the route selected, and number '0' denoted for route unselected. Hence, the selected routes for shortest path were from $S \rightarrow V1 \rightarrow V2 \rightarrow V5 \rightarrow V8 \rightarrow V11 \rightarrow V20 \rightarrow T$. In the supply or demand column, that source node, “s” was set as number ‘1’ and the sink node, t was set as number ‘-1’ because both of the nodes were the starting and the ending nodes.

Key Cell Formulas		
Cell	Formula	Copied to
F66	=SUMPRODUCT(D4:D64,F4:F64)	-
I4	=SUMIF($B$4:$B$64,$H4,$D$4:$D$64)-SUMIF($C$4:$C$64,$H4,$D$4:$D$64)	I4:I29

Table 4: Formula of the cell in figure 11

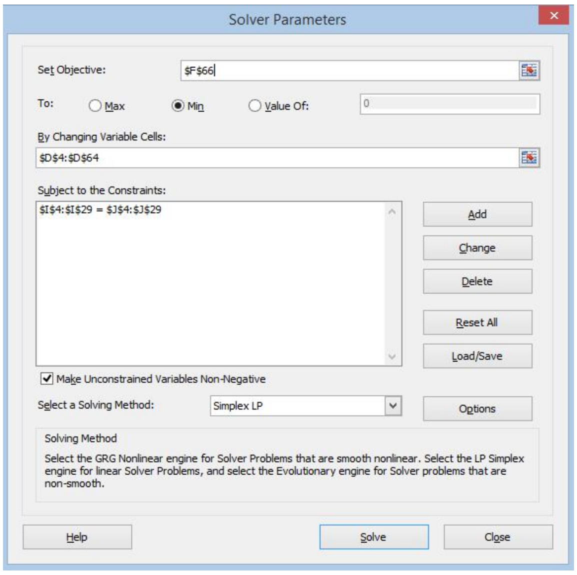

Figure 12: Solver parameter for shortest path problem in Excel

The solver parameter in Excel is shown in figure 12. The cell F66 was set as the Objective. Since the goal was to find the shortest path, therefore the minimum button was chosen in order to minimize the shortest path problem. The changing variable cells were the cells from D4 to D64. Next, the constraints of the Net Flow (cells I4 to I29) must be equal to Supply/Demand (cell J4 to J29). Before clicking on the solve button, Simplex Linear Programming was selected as the solving method. The output of the shortest path by using the excel solver would be the same as the output of the Dijkstra’s algorithm [15].

CONCLUSION

In conclusion, different number of augmenting path can be happened but maximum flow was still the same which show the same result as [5]. The overall outcome from the scope site, the maximum flow of the capacitated network graph was 4017 vehicles per hour, while JalanSepanggar and Jalan Ums were the identified bottleneck. Next, the shortest path in this weighted networkgraphwas, $S \rightarrow V1 \rightarrow V2 \rightarrow V5 \rightarrow V8 \rightarrow V11 \rightarrow V20 \rightarrow T$ viz. JalanSepanggar→JalanTuaran→JalanTuaran Bypass (North)→Jalan Lintas →Jalan Lintas →Jalan Lintas →Jalan Lintas, and it took about 21.75km from origin, IPto destination, KKIA in Kota Kinabalu. Thus,with these outputs, traffic planar couldthink of the ways to improve the identified bottlenecks, and traffic drivers could avoid the bottleneck and chose the shortest path as their desire route.

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