INFLUENCE OF CROSS-SECTIONAL AREA OF A DYNAMIC MAGNIFIER FOR VIBRATION ENERGY HARVESTING

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Abstract- Vibration energy harvesting using ambient vibrations is gaining importance day by day due to the search for alternative energy generators for small electronic devices. Cantilever beams with piezoelectric actuators mounted on it as unimorph or bimorph structures are often used for this purpose. When these systems are vibrated, the strain produced in the piezoelectric actuators will be directly proportional to the amount of energy generated. If a structure vibrates at its resonant frequency, the strain produced will be maximum and therefore more power can be generated. A Dynamic Magnifier enhances the induced strain in the energy harvesters and thereby helps to fit more modal frequencies in narrow bandwidth. In this paper, the effect of different cross-sections of the Dynamic Magnifier on vibration energy harvesting is studied; the modal frequencies of piezoelectric cantilever beam with and without magnifier are described. The results clearly showed that as the cross-sectional area decreases, the natural frequencies fall in a smaller range and the efficiency of a vibration energy harvester can be enhanced. The best cross section of the Magnifier beam is identified in the end.

Keywords- Dynamic Magnifier, Energy Harvester.

I. INTRODUCTION

The need for self-sustainable energy harvesting mechanisms has seen a lot of scope for research with the wide applications of self-powered micro-electro mechanical devices, Portable electronics, and actuation for health monitoring. One of the energy sources which is gaining much importance in this area of study is harvesting energy from ambient vibration using piezoelectric transduction mechanism.

A piezoelectric energy harvester, in its simplest form is composed of a cantilever beam with a proof mass at the tip to increase the deflection, and thereby to achieve higher strains, and hence the output power. These harvesters are most effective when operated at or close to resonant frequency, which is a difficult constraint to fulfill all the times, because the main limitation is that the ambient vibrations are present at very low frequency range (1 - 300 Hz). Therefore, Attempts have been made earlier to increase the efficiency of the energy harvesting systems: Leland and Wright and Roundy and Zhang [4] applied electrical input/axial load to actively tune the operating frequency of power harvesting device. Self-tuned systems were also proposed by Challa, Prasad, Shi and Fisher which use magnetic field to vary the stiffness of Cantilever beams.

Different configurations of Cantilever beams have been developed earlier to increase the efficiency of harvestings devices. Kong, Etrurk, and Inman [2] used impedance matching between the piezoelectric transducer and the electrical load to improve the efficiency. The problem of narrow bandwidth of ambient vibration was addressed by M. Ferrari, V. Ferrari, Guizzetti, Marioli and Taroni, [6]; they achieved the task by placing piezoelectric beam with different natural frequencies closer to each other. Badel, Guyomar, Lefevre, and Richard [7] used an electrical switching device along with harvester to maximize the output voltage of the harvester. To harvest energy from first 2 modes, an L-shaped flexible structure was proposed by Xu, Shao, Kong and Feng [8] which improved the efficiency of the harvester. Concept of Multiple masses at different locations was proposed by Lee, Yuon and Jung [9] to harvest energy from first two natural frequencies.

It is observed from the earlier studies that more number of resonant frequencies will help to generate more power from ambient vibrations. It can be possible if a Dynamic magnifier is attached to a piezoelectric cantilever beam as suggested by Aldraihem and Baz. This research is significant, however, the performance of the energy harvesting device can be further enhanced if the cross section of the beam is varied. In this paper the performance of energy harvester with a dynamic magnifier is studied for different cross sections of the magnifier beam.

II. THEORETICAL BACKGROUND

A. Beam Theory
From Euler-Bernoulli beam theory or thin beam theory, the governing equation for the undamaged free vibration of a uniform beam is given by [11],

\[ EI \frac{d^4w(x,t)}{dx^4} + \rho A \frac{d^2w(x,t)}{dt^2} = 0 \]  

(1)

Where ‘E’ is the Young’s modulus, ‘I’ is the
moment of inertia, \( w(x, t) \) is the vertical displacement of any point on the beam at a distance of \( 'x' \) from the fixed end and at time \( 't' \), \( \rho \) is the density of the beam, \( 'A' \) is the cross-sectional area. For a cantilever beam with tip mass, the characteristic equation can be written as [2]:

\[
1 + \cos \beta \cdot \cosh \beta + \frac{M_t}{\rho L} (\cos \beta \cdot \sinh \beta - \sin \beta \cdot \cosh \beta) - \frac{\beta^2 I_t}{\rho L^3} (\cos \beta \cdot \sin \beta + \sinh \beta \cdot \cos \beta) + \frac{\beta^4 M_t I_t}{\rho^2 L^4} (1 - \cos \beta \cdot \cosh \beta) = 0
\]

(2)

Where, \( \beta_n \), \( \omega_n \) is the \( n^{th} \) natural frequency of the cantilever beam, \( M_t \) is the tip mass, \( I_t \) is the mass moment of inertia of the tip mass about \( x = L \). The above equations can be used to find the any number of natural frequencies of the beam.

B. Piezoelectric Energy Harvesting
Piezoelectric materials are the materials that physically deform in the presence of an electric field, or conversely, produce an electrical charge when mechanically deformed. This effect is due to the spontaneous separation of charge within certain crystal structures under the right conditions producing an electric dipole. The constitutive equations for a piezoelectric material are given by [2]. It can be seen that the first equation is the Hooke’s Law when electric field is zero and the second is Gauss’s law of Electricity when stress is zero.

\[
\delta = \sigma / Y + dE
\]

\[
D = \varepsilon E + d\sigma
\]

(3)

Where,  
\( \delta \) - Mechanical strain  
\( \sigma \) – Mechanical Stress  
\( Y \) – Modulus of Elasticity (Young’s Modulus)  
\( d \)- Piezoelectric strain coefficient  
\( E \) – Is the electric field  
\( D \) – Electric Charge Displacement  
\( \varepsilon \) – Dielectric constant of piezoelectric material

III. PROBLEM DEFINITION

A. Cantilever with magnifier
Fig. 1 shows a cantilever beam with a dynamic magnifier, was initially suggested by Aldraihem and Baz [10] to increase the deformation of the beam while it vibrates at its first natural frequency. This was later used by Wanlu Zhou, Gopinath Reddy Penamalli, and Lei Zuo [12] who used it to damp multiple modes and showed that by optimizing few beam parameters multiple modes can be damped and deformation can be increased which in turn will lead to higher power generation using piezoelectric actuators. Piezo elements are attached on the energy harvester beam.

![Fig. 1 Cantilever Beam with Dynamic Magnifier](image)

Fig. 2. Beam with different Cross-Sections

![Fig. 2 Beam with different Cross-Sections](image)

![Fig. 3 Cantilever Beam with and without magnifier](image)

Fig. 3. Cantilever Beam with and without magnifier

B. Dynamic Magnifier with different types of cross-sections
Modal analysis of cantilever beams with different cross-sections such as rectangular, trapezoidal and triangular as shown in Fig. 2, were carried out using Ansys Mechanical software. The material for the dynamic magnifier and the beam is taken as aluminum alloy of density 2770 kg/m³, Young’s modulus of 71 GPa, and Poisson’s ratio 0.33. The first six natural frequencies are tabulated in table 1. It is to be noted that the rotational motion is ignored in the analysis.

Initially the modal analysis was carried out on a regular cantilever beam as shown in Fig. 3 with different cross-section. The analysis was carried out for cantilever beam with a magnifier also shown in Fig. 3.
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Fig. 4 shows a cantilever beam with trapezoidal cross-section and uniformly varying thickness. Modal analysis of this cantilever beam with trapezoidal cross-section was also carried out. The results are tabulated in the last column of table 1. To compare the results, bar graphs were plotted to see the variation of the frequency with the cross-section. The graphs are shown below.

IV. RESULTS AND DISCUSSIONS

Fig. 5 shows the variation of first natural frequency with the cross-section of the beam. The first 3 columns represent the variation of natural frequencies in the cantilever beam without magnifier. It is observed that as the cross-sectional area decreases, the first natural frequency and the subsequent frequencies also decreases from 7.68Hz for rectangular cross-section to 4.73 for triangular cross-section. The next three columns in Fig. 5 represent beams with a magnifier. Also, as expected [12], the first natural frequency decreases by 20% approximately for beams of all cross-sections by the addition of the magnifier which is the sole objective of attaching a dynamic magnifier.

It is observed that as the cross-sectional area decreases, the resonance occurs at a lower frequency. Even though the natural frequency for the triangular section is the lowest 4.73Hz without magnifier and 3.88Hz with magnifier, many shear modes were observed for triangular cross-section beam, which is not a desirable effect. Beam of trapezoidal cross-section on the other hand is both stable (few shear modes), when compared to triangular cross-section beam and also reduces the fundamental frequency when compared to beam of rectangular cross-section (20% drop for first mode). Hence, we can say that the trapezoidal cross-section beam is best suited for the purpose of energy harvesting.

Fig. 6 and Fig. 7 showed that the first six natural frequencies for rectangular, trapezoidal and triangular cross-sectioned cantilever beams with and without a...
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The last column of Table 1 gives the data for a trapezoidal cross-section beam with uniformly decreasing thickness which is plotted in Fig. 7. The graph shows that by reducing the thickness of the beam, the modal frequencies can be reduced further. By using a beam of uniformly reducing thickness, the natural frequency can be reduced further from 254.77Hz to 247.66Hz. This clearly shows that as the cross-sectional area decreases, the modal frequency comes down. The decrease in modal frequencies is more prominent for higher modes.

CONCLUSION

From the above analysis it is concluded that one of the effective methods to reduce the bandwidth of modal frequencies to low and narrow range is by changing its cross-sectional area. Even though the decrease in natural frequency for a beam of triangular cross-section is more than that of a beam of trapezoidal cross-section, it cannot be used effectively because of the presence of many shear modes. Hence it can be concluded that beam with trapezoidal cross-section serves the purpose better than a beam of rectangular or triangular cross-section. The bandwidth can be further reduced by using a beam of uniformly decreasing cross-section. By reducing the bandwidth to a lower and smaller range, strain produced in the system can be increased which will increase the energy harvesting capacity of the system using piezoelectric transducers.

REFERENCES


Table 2: First Six natural frequencies of a cantilever beam

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Uniform</th>
<th>Trapezoidal</th>
<th>Triangular</th>
<th>Uniform</th>
<th>Trapezoidal</th>
<th>Triangular</th>
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<tr>
<td></td>
<td>Natural</td>
<td>Magnifier</td>
<td>Frequency</td>
<td>Natural</td>
<td>Magnifier</td>
<td>Frequency</td>
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<tr>
<td>1</td>
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<td>6.63</td>
<td>5.73</td>
<td>6.35</td>
<td>6.06</td>
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<td>24.52</td>
<td>23.96</td>
<td>25.14</td>
<td>24.65</td>
<td>23.75</td>
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